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Assessing Pre-Service Mathematics Teachers' Understanding in Inductive and Deductive Reasoning in Mathematics Application

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ABSTRACT

Inductive reasoning is a key strategy for knowledge creation, problem solving, and generalization in mathematics education. Deductive reasoning, on the other hand, is the process of using logically sound methods to derive a true conclusion from a given set of premises. If the conclusion and the premises are both true, deductive validity of the conclusion is established. However, the application of inductive and deductive reasoning in teaching mathematical research at colleges of education in Ghana has not received much academic attention. Therefore, the study sort to explore pre-service teachers' understanding of inductive and deductive reasoning in teaching the mathematics (series and sequence). The research approach was mixed methods with sequential explanatory as its design. The study employed stratified and simple random sampling techniques to select a sample size of 237. The data were collected from a questionnaire administered to 237 pre-service mathematics teachers. Again, 6 pre-service mathematics teachers were selected for an interview to expand their views on inductive and deductive application of mathematics. Through the thematic analysis method, it was found that pre-service mathematics teachers perceived inductive reasoning as a process for moving from the particular to the general and deductive reasoning as general to particular and as a way to acquire mathematical knowledge through questioning. Descriptive statistics (mean, standard deviation, and coefficient of variation) were used to analyze the survey data, and the qualitative data from the respondents underwent thematic data analysis. Results indicated that, although incorporating both inductive and deductive reasoning improved mathematics teaching and learning that pre-service teachers need professional learning experiences geared towards using both inductive reasoning and deductive processes and tasks to form concepts and generalizations in mathematics. It is recommended that, policy makers provide varieties of instructional resources, including textbooks, online tools, and manipulatives, that can support pre-service teachers in implementing effective inductive and deductive reasoning activities in their teachings.

Keywords: pre-service mathematics teachers, manipulatives, inductive reasoning, deductive reasoning

INTRODUCTION

Most students enrolled in pre-university and teacher education mathematics programs most likely learnt mathematics primarily through deductive reasoning. According to Dhungana (2021), students study a variety of mathematical sciences, including algebra, geometry, data analysis, and many more, where the majority of the material is taught mostly through deductive reasoning. A fundamental type of sound reasoning is deductive reasoning. Deductive reasoning, also known as deduction, begins with a broad assertion, or hypothesis, and considers

all options in order to arrive at a particular, logical conclusion. Deductive reasoning is meant to operate on assertions of necessity or certainty as long as the premises are valid, in contrast to inductive reasoning, which leads to generalizations or probabilistic conclusions. (Evans, Thompson, & Over, 2015).

It is essential for the teaching and learning of mathematics to acquire and apply the skills of inductive and deductive reasoning in order to gain mathematical comprehension. In the scientific field of mathematics, the application of both inductive and deductive reasoning in the teaching and learning process can assist students in addressing and resolving real-world problems. Many fruitful beginnings in the history of mathematics have come from either body of knowledge. 'Mathematics is a kind of science where students can solve and tackle real-life problems by using either inductive reasoning or deductive reasoning in the teaching and learning process (Birenbaum & Rosenau, 2006). In order to improve students' ability to formulate hypotheses and generalizations from specific situations, the National Council of Teachers of Mathematics (NCTM) specified that this type of mathematical reasoning must advance in students across each educational level. Teachers in basic schools should therefore support and analyze their students, reasoning (NCTM, 2020).

The primary aim of this study is to ascertain the pre-service mathematics teachers' understanding of inductive and deductive reasoning in their coursework, whether they have acquired these reasoning techniques, how much inductive and or deductive reasoning has been taught to them in their mathematics courses, and what the benefits are of mastering these techniques for their line of work.

Statement of the Problem

Effective teaching and learning in mathematics depend heavily on pre-service teachers' grasp of inductive and deductive reasoning in mathematical applications. Even though these reasoning techniques are heavily emphasized in mathematics education, there is still a clear research gap when it comes to pre-service teachers' understanding and application of inductive and deductive reasoning concepts in practical mathematical situations such as series and sequence. Although numerous research works have examined pre-service teachers' general comprehension of reasoning techniques (Mourad, 2005; Smith, 2018; Johnson, et al., 2020), there is little empirical data that particularly addresses their comprehension of inductive and deductive reasoning in the context of applying mathematics (series and sequence). It is crucial to comprehend how pre-service teachers view and apply these reasoning techniques in mathematical problem-solving situations in order to improve development of curriculum and instructional practices in mathematics teacher education programs.

Moreover, most of the research that has been done so far has focused more on either deductive or inductive reasoning (Bellaera, Weinstein-Jones, Ilie, & Baker, 2021; Csanadi & Fisher, 2021), ignoring possible combinations and synergies between the two. Comprehensive research on pre-service teachers' concurrent mastery of both thinking techniques and their integration of them into their pedagogical practices is also required. By examining pre-service mathematics teachers understanding of inductive and deductive reasoning and its application in real-world mathematical contexts, this study seeks to close this research gap. This study aims to shed light on the cognitive processes that underlie pre-service teachers' reasoning strategies and pinpoint possible areas for instructional intervention and program enhancement in mathematics teacher preparation programs.

Research Objectives

The research sought to;

1. find out pre-service mathematics teachers' views on inductive and deductive reasoning in teaching and learning of series and sequence (mathematics).

2. determine how pre-service mathematics teachers apply inductive and deductive reasoning in real life situations in their teaching and learning of mathematics.

3. explore the pre-service mathematics teacher's perception about inductive and deductive reasoning in teaching and learning of series and sequence (mathematics).

Research Questions

1. What are the pre-service mathematics teachers' views on inductive and deductive reasoning in teaching and learning of series and sequence?

2. How do the pre-service mathematics teachers apply inductive and deductive reasoning in real life situations in their teaching and learning of mathematics?

3. How do pre-service teachers perceived inductive and deductive reasoning in teaching and learning of series and sequence (mathematics)?

LITERATURE REVIEW

The comprehension of inductive and deductive reasoning by pre-service teachers has been the subject of numerous research. For example, Csanadi and Fischer (2021) carried out research concentrating on how well instructional interventions enhance pre-service teachers' comprehension of deductive reasoning. The findings showed that specific teaching combined with real-world applications greatly improved pre-service mathematics teachers' understanding of the fundamentals of inductive and deductive reasoning.

Similarly, Komatsu and Jones (2022) investigated the use of inductive reasoning in problem-solving activities by pre-service teachers. The research revealed that although a considerable number of pre-service teachers exhibited a basic comprehension of inductive reasoning, their capacity to use it in intricate mathematics situations was restricted. This implies that there is a disconnect between pre-service teachers' theoretical knowledge and real-world application.

In contrast, Goyer (2023) investigated how pre-service mathematics teachers' education programs might incorporate both deductive and inductive reasoning. The results showed that including interesting exercises and real-world examples improved pre-service teachers' comprehension and successful application of both kinds of reasoning.

Theoretical Framework

Cognitive Constructivism

According to cognitive constructivism, people actively create their own knowledge through their interactions with the outside world, preexisting knowledge structures, and other people (Kasemsap, 2015). Pre-service teachers actively develop their knowledge of inductive and deductive reasoning in the context of reasoning by working with peers and instructors, reflecting on problem-solving techniques, and participating in mathematics assignments. When assessing pre-service teachers' reasoning skills, assessment methods influenced by cognitive constructivism place a strong emphasis on the use of genuine problems, problem-solving techniques, and metacognitive reflection (Birenbaum & Rosenau, 2006).

Mathematical Knowledge for Teaching (MKT) Framework

The specific knowledge and abilities that educators require to teach mathematics successfully are highlighted by the Mathematical Knowledge for Teaching (MKT) framework. According to this paradigm, evaluating pre-service teachers' grasp of inductive and deductive reasoning include analyzing their pedagogical content knowledge, mathematical content knowledge, and understanding of reasoning-related student thinking. Analyzing student reasoning errors, creating instructional activities that support the development of reasoning abilities, and considering the importance of reasoning in mathematical problem-solving are

some examples of assessment tasks that are aligned with the MKT framework (Nolan, Dempsey, Lovatt & O'Shea, 2015).

Situated Cognition

Situated Cognition Theory

According to the contextual cognition paradigm, learning occurs not apart from but rather inside genuine social interactions and settings. Situated cognition highlights the significance of authentic assessment problems that mirror the intricacies of teaching and learning mathematics in classroom settings when evaluating pre-service teachers' comprehension of reasoning. Analyzing pre-service teachers' reasoning practices in real-world teaching contexts, seeing how they interact with students during problem-solving exercises, and assessing how well their stated beliefs about reasoning instruction align with their actual classroom practices are some assessment approaches that draw from situated cognition (Wilson & Myers, 2000).

Socio-Cultural Theory

According to Aalto (2019), socio-cultural theory places a strong emphasis on how historical settings, cultural norms, and social interactions influence how people learn and grow (Verenikina, 2003). Socio-cultural theory emphasizes the impact of institutional frameworks, cultural norms, and cultural beliefs on the reasoning skills and instructional practices of pre-service teachers when evaluating their comprehension of reasoning. Socio-cultural theory-informed assessment techniques could look at how pre-service teachers' reasoning strategies are influenced by culture, how social interactions play a part in co-constructing reasoning knowledge, and how reasoning concepts have evolved historically in mathematics education.

METHODOLOGY

Research Paradigm

How someone sees the world through his or her lenses is the philosophical foundation of the research. A paradigm is the underlying set of assumptions that guides a researcher's technique, according to Kumatongo and Muzata (2021). The philosophical foundations of pragmatism enable and direct mixed methods researchers to employ a range of techniques to address research subjects that are no amenable to a single method.

The research paradigm utilized in this study is pragmatism because, in order to accurately address the study's research objectives, it may incorporate the use of a variety of research techniques, including action, qualitative, and quantitative research methodologies. Again, to respond to various research questions and develop the quantitative phase, this provides a clear and thorough overview of the phenomenon under study (Bryman, 2016). Additionally, it aims to gather both quantitative and qualitative information to paint a vivid picture of the issue being studied.

Research Approach

In order to gather information, and provide precise answers to research questions, the study employed a mixed methods technique. Also, for researchers to properly comprehend the problem of the study, Dawadi, Shrestha and Giri (2021) claimed that a mixed-methods approach allows the researchers to collect both qualitative and quantitative data on the phenomenon under study. Convergent parallel approach, exploratory approach, embedding approach, and explanatory approach are the four types of mixed-methods approaches that Creswell and Plano-Clark (2011) identified.

Research Design

The study used a sequential explanatory design, thus descriptive design for the quantitative phase and phenomenological design for the qualitative phase. This made it possible

for the researchers to integrate both quantitative and qualitative data in order to fully comprehend the research problem. According to Sheldon (2022), qualitative data supplied explanations and deeper insights into participants' experiences and opinions, while quantitative data revealed patterns and linkages. To supplement or further explain the quantitative findings, the researchers first gathered and examined the quantitative data, then proceeded to collect and evaluate the qualitative data. Survey questionnaires were used to determine the pre-service mathematics teachers' opinions on the use of inductive and deductive reasoning in teaching mathematics and application of deductive and inductive reasoning in real life situations after which the interview was conducted for the qualitative phase to capture the lives experience of pre-service mathematics teachers' perception on inductive and deductive reasoning in teaching and learning of mathematics.

Population

All 580 pre-service mathematics teachers at Berekum College of Education (who are under training to become future mathematics educators) in Ghana's Bono Region comprising 300 males and 280 females made up the study's population. These students were purposively selected because the researchers wanted to find out how the pre-service mathematics teachers are applying both methods on their teaching and learning process and also how they are incorporating both approaches in their teaching practice programme.

Sample and Sampling Techniques

Sampling, according to Bafarasat (2024), is the process of selecting a subset or the full population from a given sampling frame.

Based on Yamane's sample size formula, as described in Aminu, Mansur, Ya'u, Mansur, Hassan and Adam (2023), two hundred and thirty-seven (237) pre-service mathematics teachers from the population of 580 students took part in the study.

$$n = \frac{N}{1+N(e)^2},$$

where $n =$ sample size, N = population, e = the error term.

$$n = \frac{580}{1+580(0.05)^2}, \text{n} = \frac{580}{2.45}$$

$$n = 237$$

Participants were chosen using simple random sampling and convenience sampling. In order to make sure the sample was both accessible and representative, these two techniques were applied. Convenience sampling is a type of non-probability sampling in which the participants are chosen according to their availability. It is widely used in research projects, such as those done in schools, that have little funding or time constraints. Depending on their availability and willingness to participate in the study, convenience sampling was utilized in this study to select participants from particular classes. Convenience sampling is simple to perform, but it limits the generalizability of results and increases selection bias. The researchers chose to use simple random sampling in order to ensure that the sample is representative of the population.

To choose students for the sample size, the researchers employed a lottery method of simple random sampling (probability sampling) to select the respondents. In using the lottery method, a sampling frame made up of an alphabetical list of names of each student was used. The names indicated in the sampling frame were substituted with numbered cards such that each one corresponded to the name of the student. The cards were put in a box, mixed well enough, and randomly removed one by one without replacement. The number of any selected card was registered to match a student's name. This process was continued until the needed

number of respondents was attained. The method was repeatedly used in all the selected schools to select 237 students who took part in the study. This was done to guarantee that every pre-service mathematics teacher has equal chance of being chosen.

Nasu (2020) made a contribution to the field of qualitative research by stating that in order to understand 'what they are experiencing, how they interpret their experiences, and how they structure the social world in which they live,' researchers' in the field of education can always be found interviewing the people they are studying. In order to take into account experiences from the viewpoints of the informants, methodologies and processes were established through qualitative research. According to Phillippi and Lauderdale (2018), conducting qualitative research involves a conversation or interaction between the researchers and the people they are studying.

Furthermore, Merriam (2015) proposed that purposive sampling is the best appropriate method for qualitative analysis. She went on to say that this sampling strategy is predicated on the idea that the researchers must comprehend and acquire knowledge, thus it is necessary to select a sample that would allow for the greatest amount of learning. six (6) students, 3 males and 3 females were purposefully chosen for an interview for the qualitative phase.

Data Collection Instruments

The college administration gave the researchers approval before the study started. Utilizing surveys, quantitative data was obtained. In order to obtain additional data for the qualitative phase, interviews were conducted.

Questionnaire

Adapted questionnaires which have been fashioned in a Likert scale format from 1 strongly disagree through to 5 strongly agree was used to measure the constructs (Pekrun, 2011; Zimmerman, 2000). Changes in terms of wording were made to suit the local context. All the scales were pre-tested to ascertain their validity and reliability.

Noviana, and Oktaviani (2022) define a questionnaire as a set of questions distributed to individuals with the goal of collecting statistically significant information on a specific topic or event. The study used primary sources as for gathering data. Primary sources are authentic sources where the researchers directly gather data that has not been gathered before. Using a questionnaire about the variables in research objectives 1 and 2, primary data was gathered. The survey was divided into sections. The first section of the questionnaire was to collect demographic information of participants including age, gender, programme and hall of residence. The second part of the instrument the constructs under investigation. Under each construct, 10 items were used to collect responses from the participant. The other Sections which contained Likert-scale type questionnaires which were designed to collect information about preservice teachers' views and application regarding deductive and inductive reasoning techniques. The use of the questionnaire as a data collection tool allowed the researchers to quickly gather the opinions of a large number of respondents (Bryman, 2016). The reason the questionnaire was used once more was because it preserved the respondents' anonymity, allowing them to answer truthfully since their identity and confidentiality were concealed.

Interview

The purpose of interviews was to learn about participants' experiences, perceptions, beliefs, and motives. There was an interview for the qualitative stage. In example, interviews are a useful method for gathering information about participants' lived experiences for phenomenological studies (Potter & Hepburn, 2005). Semi-structured interviews were employed by the researchers to gather qualitative data for the investigation. Belina (2023) defined semi-structured interviews as those in which 'the questions are more flexibly worded

and consist of a mix of more or less structured questions. This type of interview allows the researchers' to fully explore the respondent's emerging worldview and respond to the situation as it unfolds.

According to Karahan (2022) conducting individual and focus group interviews is necessary for examining personal viewpoints because it gives researchers the contextual and anecdotal evidence, they need to comprehend these encounters. According to Chinnappan, McKenzie and Fitzsimmons (2013), people who 'not hesitate to speak, who are articulate, and who can share ideas comfortably' are the main candidates for one-on-one interviews.

In order to collect qualitative data for the study, six (6) students, three male and three female were purposefully chosen to participate in interviews According to Jarrell and Kirby (2024), for qualitative analysis, purposive sampling is most fitting. Guided by research question three, the researchers created the semi-structured interview guide. With two to ten participants, according to Boyd (2001), research saturation can usually be achieved.

The participants in the study were made fully aware of the nature of the interview and their right to withdraw at any moment by the researchers. Participants were informed that their participation in the interview was completely optional, that they may end it at any time, and that they would have the chance to ask questions before it started. The interview protocol was followed when questioning the students and recording their audio. In order to assist the study, the interview methodology comprised questions that were developed from a range of related research literature.

Data Analysis Approach

The questionnaire was administered personally and was taken back by the researchers. A total of two hundred and thirty-seven (237) questionnaires were distributed to pre-service mathematics teachers of Berekum College of Education. An introductory letter from the researchers' department was taken and sent to the authorities of Berekum College of Education to seek for permission upon which an appropriate date and time for the collection of data were discussed with the College authorities. All 237 completed questionnaires were returned.

Also, the qualitative information from the respondents were gathered from the interview to address research question 3. Qualitative data were recorded and then analysed according to principles of the qualitative method known as phenomenology, as suggested by Urcia (2021). The analysis process involved pulling phenomenological themes from the interviews and collapsing them into meaning units or broader themes. As these themes emerged, they were then transcribed and interpreted by examining the participants' common experiences with the phenomenon of care.

Validity and Reliability Evidence

All items leading to latent variables in the questionnaire were rated from 1 to 5 to assess the respondent's opinions. A Likert-scale-type questionnaire was adopted in the sense that its' psychometric scale was devised to measure and quantify the subjective preferential thinking and feelings of a subject through social interactions (Taherdoost, 2016). The validity of the questionnaires was assessed through the supervisor's judgment and also by allowing experts in the mathematics department at Berekum College of Education to look at the items in the questionnaire for their validity. These experts were employed to critically look at the content validity of the items to be convinced that the items are good to measure the construct under study. The reliability of the questionnaire items was established using a pre-test of respondents in another jurisdiction outside the study area (Alfarouq College of Education). The Cronbach's alpha of all Likert-type questionnaires from the pre-test was computed, and items with a Cronbach's alpha less than 0.7 were removed before commencing the study with the required

questionnaires. Creswell (2014) opted that a Cronbach alpha coefficient of 0.70 is considered reliable and a good indicator of internal consistency.

Reliability Test

The researchers made use of two constructs of 10 each to find out the pre-service teachers' opinions in using inductive and deductive reasoning and how they apply inductive and deductive reasoning in real life situations in teaching and learning of mathematics. The reliability of Cronbach's alpha value for all the test items concerning the constructs sought to be determined is shown in Table 1 after presenting the items to 30 respondents to assess the reliability of the questionnaires before embarking on the main study. The Cronbach's alpha of all the items was greater than 0.7, indicating their internal consistency. According to Abel, Buff and Burr. (2016), a Cronbach alpha coefficient on a scale of above 0.7 is desirable to measure construction in a study. Table 1 shows the reliability of the questions and the constructs they seek to measure.

Table 1: Reliability of Questionnaire Items Leading to Their Construct

Construct	**No. of Items**	**Cronbach's alpha (α)**
Pre-service teachers' views in using inductive and deductive	10	0.731
Pre-service teachers' real-life application of inductive and deductive	10	0.746

Source: Field Survey, 2024.

Trustworthiness of Qualitative Data

To ensure trustworthiness on the part of qualitative data, the researchers considered transferability, confirmability, credibility and dependability of the study's findings.

Transferability

This concerns the degree to which the outcomes can be employed in a wider meaning or applied to other situations. It shows how far the results of one study can be applied to other research that have similar features (De Ceunynck et al., 2017). While the goal of qualitative research is not to generalize its findings, it is believed that other researchers' who perform comparable studies with related topics will be able to benefit greatly from the findings.

De Ceunynck et al. (2017) noted that transferability is about researcher's drawing connections between the results in contexts and circumstances that are related but outside the scope of the initial study, as opposed to generalization, which asserts that the results of a specific study can be applied to all contexts relevant to the context under study.

Confirmability

The researchers' distinct point of view could skew or misrepresent the information gathered or processed. According to Coleman (2022), interpretative validity contributed to the production of objective and consistent results in cases where the study's conclusions needed to be verified by other researchers. The degree to which a researcher accurately interprets and summarizes the goals, viewpoints, and experiences of the participants is known as the study's level of interpretive validity. In order to avoid biases or predispositions, the researchers employed the reflexivity technique and engaged in critical self-reflection.

By recording personal preconceptions in memo form, the researchers were able to monitor and manage biases. The researchers continuously documented the procedure in order to provide a 'paper trail' of the processes used to verify and update the data.

In addition, the researchers diligently looked for and documented any unfavorable experiences that conflicted with earlier interviews. The degree of similarity among the student

interview answers was looked at. Stated differently, the researchers' examined the degree to which each student's interview self-report and the nominators' descriptions of them agreed.

When it came to the use of both deductive and inductive teaching methodologies, the students' self-reports were the most important factor.

Credibility

Establishing credibility involves demonstrating that the study's findings make sense from the perspective of the participants. The study's participants were the only ones qualified to assess the validity of the results because their involvement was the only way to determine how using inductive and deductive approaches affected their skill in teaching mathematics. The most important tactic for building credibility is member checks, which involve displaying research materials to research subjects (López-Zerón, Bilbao-Nieva & Clements, 2021). Participants are given the chance to indicate whether they agree or disagree with how the researchers has portrayed them.

The researchers gave the students the transcripts of the interviews so they could verify the accuracy of the material and make any required changes to fairly represent the students' opinions.

Dependability

This study discusses the stability of data across time. Dependability refers to the degree to which study findings may be repeated or replicated over an extended period of time (Jones, Gwynn, & Teeter, 2019). By rigorously adhering to the requirements while doing the research, the researchers assured reliability. Concerning their anonymity, each respondent was given guarantees. Again, the researchers collected enough data from relevant studies to back up the investigation's results and conclusions. All of the authors that were cited have been accurately cited in the reference section.

Data Analysis

Statistical Package for the Social Sciences (SPSS) version 25 was used where mean and standard deviation were used in the data's descriptive analysis. The statistical analysis did not include names or other personal data. All data were cleaned up before analysis to make sure there were no outliers (Dimitrov, 2012).

The qualitative phase was also analysed using Thematic analysis. A thematic analysis of the qualitative information from the interview was carried out to address research question 3. Qualitative data were recorded and then analysed according to principles of the qualitative method known as phenomenology, as suggested by Kumatongo and Muzata (2021). The analysis process involved pulling phenomenological themes from the interviews and collapsing them into meaning units or broader themes. As these themes emerged, they were then transcribed and interpreted by examining the participants' common experiences with the phenomenon of care. A manual transcription of interviews was used by the researchers for the Word document. This was accompanied by a study of electronic transcripts to equate them with digital records in order to guarantee accuracy. The researchers used a mobile phone recorder for interviews to help him connect personally to a computer. Each interview was handled separately, and distinct file names were given to each participant to make it easy to distinguish them. The transcripts of the interviews were sent to the interviewee through WhatsApp after they had been transcribed so that they could be reviewed and checked to see whether they accurately reflected what was meant to be referred to as the member's check. A colour-coding technique was used to identify the codes, which contained words, sentences, and phrases that held the answers to the study questions.

To identify specific themes in the text, the codes were then put into a codebook. Each group of texts was likewise assigned a code, and the themes were discovered by carefully examining each line of text and looking for any methods, connections, acts, or effects of those

activities. Codes and potential patterns have been used to establish groupings by organising and defining the main research-related associations. After concentrating on all of the transcripted interviews, the researchers compiled the data for further analysis on the broad themes of the study's questions, including relationship trends, attitudes, behaviours, and problems.

Ethical Consideration

Participants of the research work were ethically taken care of by ensuring trustworthiness in their responses in the questionnaire. Permission was taken from students who were interviewed. Also, prior notice was given concerning the date and time of the interview. To keep responses of the interviewees private, anonymity and confidentiality were assured and that interviewees responses were coded. Again, before being asked to participate, participants were made aware of the study's objectives and their rights. At any time, participants might choose not to participate in the study or to leave it without incurring any penalties. Participants' anonymity and privacy were safeguarded. Any personal data that might be used to identify them was kept private. All participants gave their informed consent to participate in the study voluntarily, no one was forced or coerced. They gave their permission and participated freely. Participants were handled with dignity and respect by the researchers. Any actions or inquiries that might be harmful or uncomfortable were avoided. For fairness, regardless of their origin or personal traits, participants received fair and equal treatment. No participant received special treatment or experienced any form of discrimination.

RESULTS AND DISCUSSION

Demographic Data of Participants

Table 2 presents the results of participants' demographic information.

In all, 237 pre-service mathematics teachers participated in the study. This number comprised 136 males, representing 57.4%, and 101 females, representing 42.6%. The age distribution of students ranged between 20 - 22 years, 23 - 25 years and 26 years and above were respectively 56, 118 and 63. These values accounted for 23.6%, 49.8% and 26.6% accordingly. The survey also involved 83 Buchanan Hall, 57 Nicholas Hall, 60 Steward Hall and 57 Yiadom Boakye Hall pre-service mathematics teachers which represented 26.6%, 24.1%, 25.3% and 24.1% respectively of the total number of participants.

Table 2*:* Demographic Data

Demographics	**Frequency (N)**	**Percentages (%)**
Gender	***237***	***100***
Male	136	57.4
Female	101	42.6
Age	***237***	***100***
20-22 years	56	23.6
23-25 years	118	49.8
26 years and above	63	26.6
Hall of Affiliation	**237**	**100**
Buchanan	*63*	*26.6*
Nicholas	*57*	*24.1*
Steward	*60*	*25.3*
Yiadom Boakye	*57*	*24.1*

Source: Field Data, 2024.

Presentation of Findings

The study's findings are reported in this section and are organized in accordance with the research questions.

Research Question 1: What are the pre-service mathematics teachers' views on inductive and deductive reasoning in teaching and learning of series and sequence?

To assess the pre-service mathematics teachers' views on inductive and deductive reasoning in teaching and learning of series and sequence(mathematics), the respondents were asked to rate 5-point Likert scale item with 1 showing least rating and 5 showing strong. For analysis purposes, the mean, standard deviation and the co-efficient of variation of the responses given by the respondents were computed. The results of the respondents' responses on pre-service mathematics teachers views and understanding on inductive and deductive reasoning in teaching and learning of series and sequences were analysed with mean ranks. The responses from respondents were presented in Table 3.

Table 3: Descriptive Statistics of pre-service mathematics teachers' views (understanding) on inductive and deductive reasoning in teaching and learning of series and sequence

Variables	Mean	Std. Dev.
I believe integrating both inductive deductive reasoning should be emphasized more in basic school mathematics curriculum.	4.3882	0.6453
I incorporate both inductive and deductive reasoning in my lesson plans when teaching mathematics.	4.3038	0.8237
I am able to articulate the steps involved in inductive and deductive reasoning to solve mathematical problems.	4.2321	0.7018
I understand the difference between inductive and deductive reasoning.	4.2152	0.6445
I feel prepared to integrate activities and lessons that develop students' skills in inductive and deductive reasoning into my future mathematics teaching practice.	4.1983	0.7121
I can effectively apply both inductive and deductive reasoning to prove mathematical statements and theorems.	4.177	0.7028
I believe that inductive and deductive reasoning helps to foster creativity and critical thinking in mathematics.	4.1477	0.7006
I believe that understanding inductive and deductive reasoning is crucial for pre-service mathematics teachers.	4.0253	0.7698
I feel that my pre-service education adequately prepared me to teach inductive and deductive reasoning in mathematics.	3.9409	1.0357
I believe that understanding inductive and deductive reasoning is crucial for students to grasp mathematical concepts**.**	3.7975	1.1937
Grand mean and standard deviation	**4.1426**	**0.7955**

Source: Field Data, 2024.

The results as depicted in Table 3 revealed that integrating both inductive deductive reasoning should be emphasized more in basic school mathematics curriculum was the most dominant measure of the pre-service mathematics teachers' views on inductive and deductive reasoning in mathematics. It attained a mean of 4.3882, with a standard deviation of .6453 showing the homogeneity of views expressed by the respondents. I incorporate both inductive and deductive reasoning in my lesson plans when teaching mathematics was the second most influential measure. It gained a mean score of 4.3038 and a standard deviation value of .8237

indicating common views expressed by the respondents. I am able to articulate the steps involved in inductive and deductive reasoning to solve mathematical problems was rated third as a measure. It secured a mean of 4.2321 and a standard deviation of .7018 which signified the respondents expressed similar opinions.

Additionally, I understand the difference between inductive and deductive reasoning was the next rated response by the respondents. It secured a mean score of 4.2152 and a standard deviation value of 0.6445, showing a common opinion shared by the respondents. Again, I feel prepared to integrate activities and lessons that develop students' skills in inductive and deductive reasoning into my future mathematics teaching practices was next measure, which secured a mean score of 4.1983 and the standard of 0.7121. I can effectively apply both inductive and deductive reasoning to prove mathematical statements and theorems was rated sixth by the respondents with a mean of 4.1772, standard deviation of 0.7028 which also indicate that the respondents shared similar opinion on that latent variable. The least rated variable for this construct was, I believe that understanding inductive and deductive reasoning is crucial for students to grasp mathematical concepts with the mean 3.7975 and the standard deviation of 1.1937.

The grand mean for the pre-service mathematics teachers views on inductive and deductive reasoning was 4.1426 with a corresponding standard deviation of 0.7955 and a coefficient of variation of 19.21% showing a very strong homogeneity of views expressed by the respondents. This further indicated that, the pre-service teachers were of the opinion that incorporating both inductive and deductive reasoning in teaching and learning of mathematics will enhance their confidence levels, improve their performance and make teaching and learning more practical. This result is also consistent with study conducted by Goyer (2023) which revealed that incorporating both inductive and deductive reasoning improved pre-service teachers' comprehension and successful application of both kinds of reasoning.

Research Question 2: How do the pre-service mathematics teachers application of inductive and deductive reasoning in real life situations in their teaching and learning of mathematics?

Again, the respondents were asked to rate 5-point Likert scale items measuring students' assessments on how apply inductive and deductive reasoning in real life situations in their teaching and learning of mathematics, with 1 representing the least rating and 5 representing a strong rating. The mean, standard deviation and coefficient of variation of the respondents' responses were calculated for analysis purposes. The mean ranks were used to analyse the responses. The findings and the analysis are displayed in Table 4.

Table 4: Descriptive Statistics of pre-service mathematics teachers apply inductive and deductive reasoning in real life situations in their teaching and learning of mathematics

Variables	Mean	Std. Dev.
Pre-service mathematics teachers who incorporate both inductive and deductive reasoning techniques into their lessons are better equipped to address diverse learning styles among students.	4.5063	0.5721
The application of inductive and deductive reasoning by pre-service mathematics teachers encourages active student participation and engagement in the learning process.	4.2489	0.8395
I find it beneficial when pre-service mathematics teachers apply both inductive reasoning to guide students towards discovering mathematical patterns in mathematics.	4.2363	0.6469

I believe pre-service mathematics teachers should receive training specifically focused on incorporating inductive and reasoning techniques into their teaching methods.	4.1308	1.0104
Pre-service mathematics teachers should be encouraged to use both inductive and deductive reasoning to derive conclusions from established mathematical principles.	4.0549	0.9484
Real-life applications provided by pre-service mathematics teachers during lessons help students grasp abstract mathematical concepts more effectively.	3.9325	1.0731
Pre-service mathematics teachers should prioritize teaching inductive and deductive reasoning skills alongside mathematical concepts to foster critical thinking among students	3.9072	1.1199
I believe pre-service mathematics teachers should receive ongoing support and resources to effectively integrate both inductive and deductive reasoning into their teaching practices.	3.9030	1.0226
Real-life scenarios presented by pre-service mathematics teachers help students recognize the relevance of mathematical concepts in their daily lives.	3.8608	1.1542
I believe that incorporating real-life examples in teaching mathematics helps pre-service teachers understand the concept better.	3.7975	1.2043
Grand mean and standard deviation	**4.0578**	**0.9598**

Source: Field Data, 2024.

The results in Table 4 indicate that pre-service mathematics teachers who incorporate both inductive and deductive reasoning techniques into their lessons are better equipped to address diverse learning styles among students was adjoined the most influential measure as it obtained an outstanding mean of 4.5063 with standard deviation, 0.5721 indicating a very strong opinion expressed by the respondents. The application of inductive and deductive reasoning by pre-service mathematics teachers encourages active student participation and engagement in the learning process was rated second by the respondents with mean 4.2489, standard deviation of 0.83947. I find it beneficial when pre-service mathematics teachers apply both inductive reasoning to guide students towards discovering mathematical patterns in mathematics was the next rated measure with mean 4.2363 with a standard deviation of 0.6469 also indicating the similar views expressed by the respondents. Also, I believe pre-service mathematics teachers should receive training specifically focused on incorporating inductive and reasoning techniques into their teaching methods was rated fourth which secured a mean of 4.1308 and the standard of 1.0104. Again, Pre-service mathematics teachers should be encouraged to use both inductive and deductive reasoning to derive conclusions from established mathematical principles was the fifth rated latent variable with mean 4.0549 and a standard deviation of 0.9484. The grand mean of pre-service mathematics teachers applying inductive and deductive reasoning in real life situations in their teaching and learning of mathematics was 4.0578 with a corresponding standard deviation of 0.9598. To confirm the views expressed by the respondents, the coefficient of variation was calculated and it was found to be 23.65% which shows a strong homogeneity of opinions expressed by the respondents. The results of this study support Jones et al. (2020), who investigated the use of inductive reasoning in problem-solving activities by pre-service teachers. Their research revealed that, a considerable number of pre-service teachers exhibited a basic comprehension of inductive reasoning. The findings also support the work done by Csanadi and Fischer (2021) on how well instructional interventions enhance pre-service teachers' comprehension of deductive

reasoning. The findings showed that specific teaching combined with real-world applications greatly improved their understanding of the fundamentals of deductive reasoning.

Research Question 3: How do pre-service mathematics teachers perceived inductive and deductive reasoning in teaching and learning of series and sequence (mathematics)?

To address research question three, semi-structured interviews were conducted using the qualitative research paradigm approach. In order to give the participants' lived experiences on how they perceive inductive and deductive reasoning in mathematics, a phenomenology study design was used. The phenomenology offers clarification or a thorough comprehension of the problem. Six purposefully chosen pre-service teachers made up the population for this phase, were asked to give their thoughts on inductive and deductive reasoning in mathematics instruction. The study employed purposive and theory or concept-based sampling strategies. Because the study participants were well-informed and had prior experience learning mathematics through both inductive and deductive reasoning, a concept- or theory-based technique was chosen for this study.

The researchers created a semi-structured interview as the instrument to gather data on students' perceptions of deductive and inductive reasoning in mathematics education. The third study objective was taken into account when designing the semi-structured. Six non-study participants from Alfarouq College of Education in the Bono Region participated in a pre-test of the semi-structured interview guide. Prior to the main study, a pre-testing phase was conducted to determine how pre-service mathematics teachers understanding and views affect students' perceptions of mathematics instruction and learning through deductive and inductive reasoning. Luckily, there were no issues discovered during the pre-testing, therefore no revisions were made. In order to address ethical concerns, the students' consent was obtained through direct communication, and they were given the guarantee that they could leave the study at any time if it made them uncomfortable. Interview dates and times were decided upon based on each student's availability and convenience. By telling participants what the goal of the study was, the researchers' demonstrated transparency and honesty with the subjects. The researchers' set up an interview with the students in the classrooms where they receive one-on-one macro instruction.

Experts in the departments of mathematics and information technology were given a semi-structured interview guide to review and provide feedback on items related to clarity, ambiguity, relevance, and generality in order to guarantee the validity and trustworthiness of the process. The investigator additionally guaranteed that ethical protocols, including confidentiality, anonymity, and voluntary involvement, were meticulously observed throughout the data gathering procedure. The investigator made certain that the data gathering procedure was taking place in an authentic and organic setting. The same questions were posed to the participants in the semi-structured interview, but in a different order. In order to enhance validity, participant review and participant verbatim narratives were combined with other data collection techniques. In order to ensure accurate depiction, the researchers' invited participants to check her synthesis of the participant interviews. In order to confirm that the information recorded appropriately reflected their opinions or positions, the participants were required to read the transcript of the discussion. To provide verbatim reports of the events in the interview session material for reliability checks, audio recordings of the interviews were made during the process.

The researchers manually entered text into the audio recordings to create a transcription as soon as the interview was over. In doing so, it was possible to get the participants' verbatim accounts and eliminate any statements that overlapped. Based on similar patterns identified by the study's themes, the replies were classified, described, and categorized. Furthermore, the data revealed relationships and linkages that helped to reinforce the study's themes through

narrative conversations. To preserve anonymity, the date and informant identity or code were recorded in the verbatim presentation.

The semi-structured interview data was analyzed using the thematic analysis approach. After that, the researchers transcribed each semi-structured interview response and categorized them into relevant themes that aligned with the study's goals. Patterns and correlations were taken into consideration when grouping and paraphrasing the responses. To bolster the themes, several responses were also written exactly as written.

The study's research questions guided the presentation and discussion of the study's findings. Through conversations, the results are also contrasted with the body of current literature.

The research participants were asked to share their views on how inductive reasoning helped them to understand mathematical concepts and students shared the following thoughts which were captured as the themes: mathematical patterns and relationships, explore mathematical concepts and develop critical thinking skills.

On the theme of inductive reasoning helping students to discover their mathematical patterns and relationships, explore mathematical concepts, develop critical thinking skills, the results of the semi-structured interviews established that the respondents hold the view that incorporating inductive reasoning in teaching and learning of mathematics help them to develop their critical thinking skills, mathematical patterns and relationship and also builds their mathematical concepts.

Some of them confirmed that when inductive reasoning is used in mathematics lessons, they feel very confident, while others express the contrary. For instance, when the respondents were asked how they perceive the role of inductive reasoning in helping them to understand mathematical concepts, the following responses were given by some of them:

'Inductive reasoning allows me to discover mathematical patterns and relationships on my own, but the process is very long'

'Inductive reasoning encourages me to explore mathematical concepts through hands-on activities and experimentation.'

'Inductive reasoning helps me develop critical thinking skills by challenging me to make logical inferences from observed data.' *(Interviewed Data, 2024)*

These results agree with those inferred by Mourad (2005). The results demonstrated how poorly learners could translate between different representations. Furthermore, when provided a table representation of the situation, students were most successful in expressing it with other representations; nevertheless, when given an algebraic representation, they struggled to translate it to other representations. Results from the study also supports the study done by Smith (2018), Bellaera, Weinstein-Jones, Ilie, and Baker (2021) also indicated that, understanding patterns, developing hypotheses, and making predictions in mathematics all depend on inductive reasoning.

Similarly, on the theme of designing of activities for students, providing sufficient guidance to students and finding suitable resources or examples, the findings of the semi-structured interviews revealed that incorporating inductive reasoning in teaching and learning has some challenges such as designing of activities for students, providing sufficient guidance to student, finding suitable resources or examples. The findings of the semi-structured interviews revealed that there were challenges pre-service mathematics teachers face in teaching and learning of mathematics through inductive reasoning.

When prompted, the respondents shared with the researchers their feelings about the challenges they face when they incorporate inductive reasoning in their teachings. The following answers were some of them provided:

'One challenge is designing activities that effectively engage students in the process of inductive reasoning.'

'Another challenge is providing sufficient guidance to students without stifling their independent thinking during inductive reasoning tasks.'

'Some pre-service teachers may struggle to find suitable resources or examples to support inductive reasoning instruction.' *(Interviewed Data, 2024)*

The findings of the semi-structured interviews revealed that the respondents encountered some challenges that pre-service mathematics teachers experienced when teaching through inductive reasoning. The study is in line with research done by Komatsu and Jones, (2022) who investigated the use of inductive reasoning in problem-solving activities by pre-service teachers. The research revealed that although a considerable number of pre-service teachers exhibited a basic comprehension of inductive reasoning, their capacity to use it in intricate mathematics situations was restricted. This implies that there is a disconnect between pre-service mathematics teachers' theoretical knowledge and real-world application.

Regarding the theme of deductive reasoning helping learners to develop logical thinking skills, critically analyze mathematical statements, and promoting mathematical thinking, the findings of the semi-structured interviews established that the respondents hold the view that incorporating deductive reasoning in mathematics lessons has been very beneficial to the pre-service mathematics teachers. They went further to say that they felt comfortable and confident when using deductive reasoning, which has greatly improved their confidence in teaching of mathematics. The following answers were given by the respondents when asked, 'What are the main benefits, in your opinion, of teaching mathematics through deductive reasoning'?

'Deductive reasoning helps me develop logical thinking skills and understand the structure of mathematical arguments and proofs and feel very comfortable with it.'

'Using deductive reasoning cultivates my ability to critically analyze mathematical statements and recognize valid reasoning, builds my confidence.'

'Deductive reasoning promotes precision and rigor in mathematical thinking by requiring me to justify my conclusions systematically.' *(Interviewed Data, 2024).*

The findings also support the work done by Csanadi and Fischer (2021) on how well instructional interventions enhanced pre-service teachers' comprehension of deductive reasoning. The findings showed that specific teaching combined with real-world applications greatly improved students understanding of the fundamentals of deductive reasoning.

Again, looking at themes from the respondents about the main challenges that pre-service mathematics teachers face when teaching and learning using deductive reasoning, the following themes were observed: abstract concepts, deductive proofs, struggle to make deductive reasoning tasks engaging or relevant to students' everyday experiences.

The following responses were provided by the respondents when asked to share their opinions about the main challenges they encountered about the deductive reasoning in teaching and learning of mathematics.

'One challenge is explaining abstract concepts or mathematical principles in a way that students can understand and apply deductive reasoning.'

'Another challenge is guiding students through the process of constructing deductive proofs and helping them avoid logical fallacies.'

'Sometimes I struggle to make deductive reasoning tasks engaging or relevant to students' everyday experiences.' *(Interviewed Data, 2024).*

The findings were also not different from the study conducted by Johnson et al. (2019) who conducted a comparative analysis of pre-service mathematics teachers' performance on inductive and deductive reasoning tasks. The study found that while participants generally performed better on inductive reasoning tasks, their proficiency in deductive reasoning varied significantly, indicating the need for targeted instructional support in deductive reasoning.

In addition, the following themes were recorded after the interview on how pre-service teachers perceive the effectiveness of incorporating both inductive and deductive reasoning in

mathematics instruction: comprehensive approach and different learning styles, deeper understanding of mathematical concepts and problem-solving abilities. The findings of the semi-structured interviews showed that the respondents have a very strong opinion that incorporating both inductive and deductive reasoning in their teaching and learning will help the students to develop different learning styles, develop a deeper understanding of mathematical concepts and enhances their problem-solving abilities and allowing students to experience the richness of mathematical reasoning and its applications.

The following responses were provided by some of the respondents when asked to share their opinions about how they perceive the effectiveness of incorporating both inductive and deductive reasoning in mathematics instruction.

'I believe that integrating both inductive and deductive reasoning provides a more comprehensive approach to teaching mathematics, catering to different learning styles.'

'Using a combination of inductive and deductive reasoning helps students develop a deeper understanding of mathematical concepts and enhances their problem-solving abilities.'

'Balancing inductive and deductive reasoning instruction allows students to experience the richness of mathematical reasoning and its applications.' *(Interviewed Data, 2024).*

The outcome of their opinions was in line with cognitive constructivist theory which posits that, pre-service teachers actively develop their knowledge of inductive and deductive reasoning in the context of reasoning by working with peers and instructors, reflecting on problem-solving techniques, and participating in mathematics assignments. When assessing pre-service teachers' reasoning skills, assessment methods influenced by cognitive constructivism place a strong emphasis on the use of genuine problems, problem-solving techniques, and metacognitive reflection (Birenbaum & Rosenau, 2006).

Finally, the following themes were emerged when the pre-service teachers were asked about support needed to effectively incorporate inductive and deductive reasoning into their teaching practice: professional development opportunities, 'Mentorship, instructional resources, including textbooks, online tools, and manipulatives. The findings of the semi-structured interviews showed that, the pre-service mathematics teachers were of a strong convection that, when adequate support such mentorship training, provision of adequate teaching and learning resources are provided to them, it will make the incorporation of both reasoning very effective in teaching and learning of mathematics. The following were the responses some of them provided.

'Pre-service teachers would benefit from professional development opportunities focused on designing inquiry-based lessons and constructing deductive proofs.'

'Mentorship from experienced educators who can provide guidance and feedback on inductive and deductive reasoning instruction would be valuable.'

'Access to a variety of instructional resources, including textbooks, online tools, and manipulatives, can support pre-service teachers in implementing effective inductive and deductive reasoning activities.' *(Interviewed Data, 2024).*

The findings collaborate with Situated Cognitive Theory which revealed that, analyzing pre-service teachers' reasoning practices in real-world teaching contexts, seeing how they interact with students during problem-solving exercises, and assessing how well their stated beliefs about reasoning instruction align with their actual classroom practices are some assessment approaches that draw from situated cognition (Wilson & Myers, 2000).

CONCLUSION

The research's findings showed that pre-service mathematics teachers had a positive thought about the inductive and deductive reasoning in mathematics. This suggests that

incorporating both reasoning in mathematics curriculum will go a long way to improve the performance of students and also contribute to effective teaching and learning of mathematics at all levels of education.

Similarly, applying inductive and deductive reasoning in real-life situations make mathematics teaching and learning more practical and interesting as pre-service mathematics teachers who incorporate both reasoning methods are better equipped to address diverse learning styles among students.

Again, it also came to light that there were some challenges encountered by pre-service mathematics teachers in incorporating both inductive and deductive reasoning. The study's findings from the interview also revealed that some pre-service teachers struggled to find suitable resources or examples to support inductive reasoning instruction. Another challenge was provision of instructional resources, including textbooks, online tools, and manipulatives, that can support pre-service teachers in implementing effective inductive and deductive reasoning activities in their teachings.

RECOMMENDATION

Based on the results of the study, the following recommendations are being proposed. Integrating both inductive and deductive reasoning provide a more comprehensive approach to teaching mathematics, catering to different learning styles, using a combination of inductive and deductive reasoning helps students develop a deeper understanding of mathematical concepts and enhances their problem-solving abilities so it recommended that Ghana Education Service should incorporate both inductive and deductive reasoning methods in basic school curriculum.

Again, balancing inductive and deductive reasoning instruction allows students to experience the richness of mathematical reasoning and its applications so it is recommended that pre-service mathematics teachers incorporate both techniques into their teaching and learning to be better equipped to address diverse learning styles among themselves and their students.

Finally, policy makers should provide varieties of instructional resources, including textbooks, online tools, and manipulatives, that can support pre-service teachers in implementing effective inductive and deductive reasoning activities in their teachings.

STATEMENTS AND DECLARATIONS

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Ethical Approval

A letter from the authors department was issued to the authorities of Berekum College of Education (where the research was carried out) requesting permission to undertake the study.

The privacy and anonymity of the participants were respected. Again, to ensure that the study is conducted in an ethical and responsible manner, researchers adhered to specified ethics when doing mixed methods research. Considerations included informed consent, privacy and secrecy, fair treatment, openness, and authorization. First, the participant was briefed by the researchers on the purpose, methodology, potential threats, and advantages of the study. Additionally, participants were made aware that they could leave the study any time they feel uncomfortable.

Authors Contribution

Conceptualization and methodology, Prince Kusi; validation, Ebenezer Bonyah and Prince Kusi; data entry and analysis, Emmanuel Teku, Prince Kusi and Sampson Kwame Effah; editing, visualization, and supervision, Ebenezer Bonyah, Sampson Kwame Effah and Emmanuel Teku. All authors have read and agreed to the published version of the manuscript.

Conflict of Interest

The authors declare no conflict of interest.

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