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Dynamic Behavior Prediction of Magnetorheological Fluid Dampers using Neural Networks

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ABSTRACT

Parametric and non-parametric models have been studied to predict the behavior of magnetorheological (MR) fluid dampers by a lot of researchers due to their nonlinear dynamics. In this paper, the direct and inverse dynamic identification for MR fluid dampers using recurrent neural networks are investigated to demonstrate the more accurate and efficient model. The effect of neural networks construction on the prediction quality of dynamic performance of MR damper is introduced in details. The trained direct identification neural network model can be used to predict the damping force of the MR fluid damper on line and the inverse dynamic neural network model can be used to generate the command voltage applied to the damper coil through supervised learning. The architectures and the learning techniques of direct and inverse neural network models for MR fluid dampers are introduced and simulation results are discussed. Finally, the trained neural network models are used to predict the damping force of the MR fluid damper accurately and precisely. Moreover, validation results for the neural network models are proposed and used to evaluate their performance. Validation results with several data sets indicate that the proposed direct and inverse identification models using recurrent neural networks can be used to predict the dynamic performance of MR fluid dampers perfectly.

Keywords: Magnetorheological Dampers, Neural Networks, Nonparametric Identification, Nonlinear Systems

INTRODUCTION

There are a lot of potential applications for magnetorheological (MR) dampers in many engineering systems related to vibration control and mitigation. They have great controllable capacities, fast responses, and low power requirements (Turcotte, East, & Plante, 2022). Modelling MR dampers is a very important technique to capture the real behavior and it's so useful in their control systems. An adequate model of MR dampers and prediction accuracy are needed to achieve an adaptable semi-active control system. MR dampers have been implemented in several engineering applications such as cars (Shehata Gad & El-Demerdash, 2022; Karkoub & Zribi, 2006), trains (Liao & Wang, 2003), airplanes (Luong, Jang, & Hwang, 2020) and civil structures (Turcotte, East, & Plante, 2022; Bani-Hani & Sheban, 2006).

The dynamic behavior of MR dampers was studied in numerous research publications during last decades to investigate their responses and electromagnetic effects using neural networks, for example (Metered, Bonello, & Oyadiji, 2010; Khalid, 2014; Liao & Wang, 2005). Direct and inverse approaches are mainly the two ways to formulate MR dampers models. For direct models, a proper force-voltage or force-current model of the MR damper is formulated to predict the damping force as a function of voltage/current and displacement across the damper. Although, the changing of the applied voltage/current is unspecified when used in real engineering systems, which produces complexity in covering most applications to determine the real experimental conditions. Sufficient training data and knowledge of the MR damper variables are highly required to generate a very good prediction accuracy of damper performance (Metered, Bonello, & Oyadiji, 2010; Liao & Wang, 2005). For inverse models, a suitable voltage-force or current-force model of the MR damper is constructed to predict the voltage/current as a function of damping force and displacement across the damper, which is suitable for most engineering applications.

The modeling process is not easy due to the nonlinearity and dynamic response of MR dampers. The dynamic behavior of MR dampers depends on both the magnitude of the magnetic field generated by the applied voltage/current and the piston movement (Ma et al., 2002; Wang et al., 2003; Xiao Qing Ma, Rakheja, & Su, 2007). During recent decades, a lot of models have been published based on mathematical equations (Wang & Liao, 2011; Sahin, Engin, & Çesmeci, 2010) and experimental work (Yang, 2001; Dyke et al., 1998; Yao et al., 2002) to predict the nonlinear and hysteretic phenomena of MR dampers. Several non-parametric models have been efficiently formulated to improve the prediction accuracy further and map the relationship between the damping force and input voltage/current directly. Neural networks (NNs) have been broadly used due to their accepting fitting and learning capabilities to model the nonlinearity of MR dampers. Firstly, a multilayer perceptron was made to capture the inherent nonlinear behavior of MR dampers and published by Chang and Roschke (1998). Also, a static model was proposed for two types of MR dampers using neural networks and the generated results scaled to describe the nonlinear properties (Tudón-Martínez et al., 2012). Furthermore, a feedforward neural network and recurrent neural were utilized successfully to produce accurate models for MR dampers as done by Metered, Bonello, and Oyadiji (2010), Liao and Wang (2005). Nonlinear autoregressive networks (Bittanti, Savaresi, & Montiglio, 2004), radial basis function networks (Du, Lam, & Zhang, 2006), and recursive lazy learning (Boada et al., 2011) have also been presented in recent years. The above-mentioned research papers have concluded that the neural-based modeling are very successful methods for predicting the dynamic behavior and hysteresis characteristics of MR dampers.

In this paper, a parametric study of both feedforward neural network (FNN) and recurrent neural network (RNN) is introduced to model the direct and inverse dynamics of MR dampers, for the first time, based on the modified Bouc-Wen model (Spencer et al., 1997). The rest of this paper is organized as follows: The modified Bouc– Wen model for MR fluid dampers is described next section to generate the training data. Section 3 deals with direct identification of MR fluid dampers using NN. Both the direct and invers models are proposed for modeling MR fluid dampers, and characteristics of these two models are discussed and compared. In section 4, the direct NN for modeling is explored. In section 5 the inverse dynamics of MR fluid dampers is explored. The performances of the NN for the MR fluid dampers are validated. In section 6, conclusions and discussions on future work are presented.

MODELLING OF THE MR DAMPER CHARACTERISTICS

In this study, two types of modelling were used to compare parametric and nonparametric models. The parametric model used the modified Bouc–Wen model proposed by Spencer et al. (1997) for impact loading applications, while the nonparametric used a proposed neural network model. Both models were used for prediction of MR damper behaviour and for further simulation analysis, to evaluate MR damper performance with a control strategy. The schematic diagram of the impact loading system for developing the modified Bouc–Wen model.

The Modified Bouc–Wen Model of the MR Fluid Damper

Figure 1 depicts a mechanical idealization of an MR fluid damper that has been demonstrated to properly anticipate the behaviour of the MR fluid damper across a wide range of inputs. The following equations control the phenomenological model developed by Spencer et al. (1997):

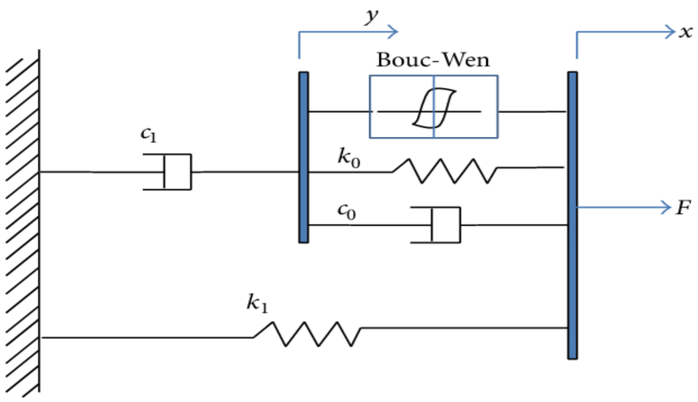


Figure 1: The dynamic model for the MR fluid damper

$$F = c_1\dot{y} + k_1(x - x_0) \quad (1)$$

$$\dot{y} = \frac{1}{c_0 + c_1}\{\alpha z + c_0\dot{x} + k_0(x - y)\} \quad (2)$$

$$\dot{z} = -\gamma|\dot{x} - \dot{y}||z|^{n-1}z - \beta(\dot{x} - \dot{y})|z|^n + \delta(\dot{x} - \dot{y}) \quad (3)$$

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u \quad (4)$$

$$c_1 = c_1(u) = c_{1a} + c_{1b}u \quad (5)$$

$$c_0 = c_0(u) = c_{0a} + c_{0b}u \quad (6)$$

$$\dot{u} = -\eta(u - v) \quad (7)$$

where x and F are the displacements and the force generated by the MR fluid damper respectively; y is the internal displacement of the MR fluid damper; u is the output of a first-order filter and v is the command voltage sent to the current driver. In this model, the accumulator stiffness is represented by k1; the viscous damping observed at large and low velocities are represented by c0 and c1, respectively. k0 is present to manage stiffness at high speeds, whereas x0 is utilized to account for the accumulator's influence. For the Bouc–Wen model, is a scaling value. The scale and shape of the hysteresis loop can be adjusted by γ, β, δ, and n. A total of 14 model parameters, which are shown in **Table** 1. To train the proposed neural networks, appropriate training data sets are required. The training data sets should cover most situations of practical applications to let the trained neural network models predict well while at the same time the selected data sets should be simple to speed up the training process. The selected data sets to be used to train the neural network models for MR fluid dampers are illustrated in **Table** 2, in which the validation and test data sets for the network training are also shown. In **Table** 2, the displacement input is a Gaussian white noise signal, the command voltage input consists of different signals within different time intervals, and the force is produced by the modified Bouc–Wen model according to the displacement and command voltage inputs. The time intervals of signals used for training, validation, and testing of NN are listed in Table 2, which are produced using the modified Bouc–Wen model described by equations (1)–(7) and the parameters given in **Table** 1, are shown in Figure 1. The input and output data sets are produced at a time increment of 0.002 s and can be accessed as vectors x, v, F, and their combinations.

Table 1: Parameters for the model of MR fluid damper

Parameter	Value	Parameter	Value
$c0a$	784 N s m^{-1}	α_a	12441 N m^{-1}
$c0b$	1803 N s V^{-1} m^{-1}	α_b	38430 N V^{-1} m^{-1}
k_0	3610 N m^{-1}	γ	136320 m^{-2}
$c1a$	14649 N s m^{-1}	β	2059020 m^{-2}
$c1b$	34622 N s V^{-1} m^{-1}	δ	58
k_1	840 N m^{-1}	n	2
x_0	0.0245 m	η	190 s^{-1}

Table 2: Training test data set

Signals	Time interval (sec)				
	0 - 30	30 - 35	35 - 40	40 - 45	45 - 50
Displacement	GWN[a]				
Voltage	GWN[b] + 2.5	5	2.5	0	2.5+2.5sin(4πt)
Force	Produced by modified Bouc-Wen model				

Note: a – Gaussian white noise (frequency: 0-3 Hz; amplitude: ± 0.02 m);
b – Gaussian white noise (frequency: 0-4 Hz; amplitude: ± 2.5 V).

Model Training

Figure 2 depicts the time histories of the training data sets (**Table 2**) generated using the modified Bouc–Wen model defined by equations (1)–(7) and the settings listed in **Table 1**. The input and output data sets are generated in 0.002 second intervals and are available as vectors x, v, F, and their combinations.

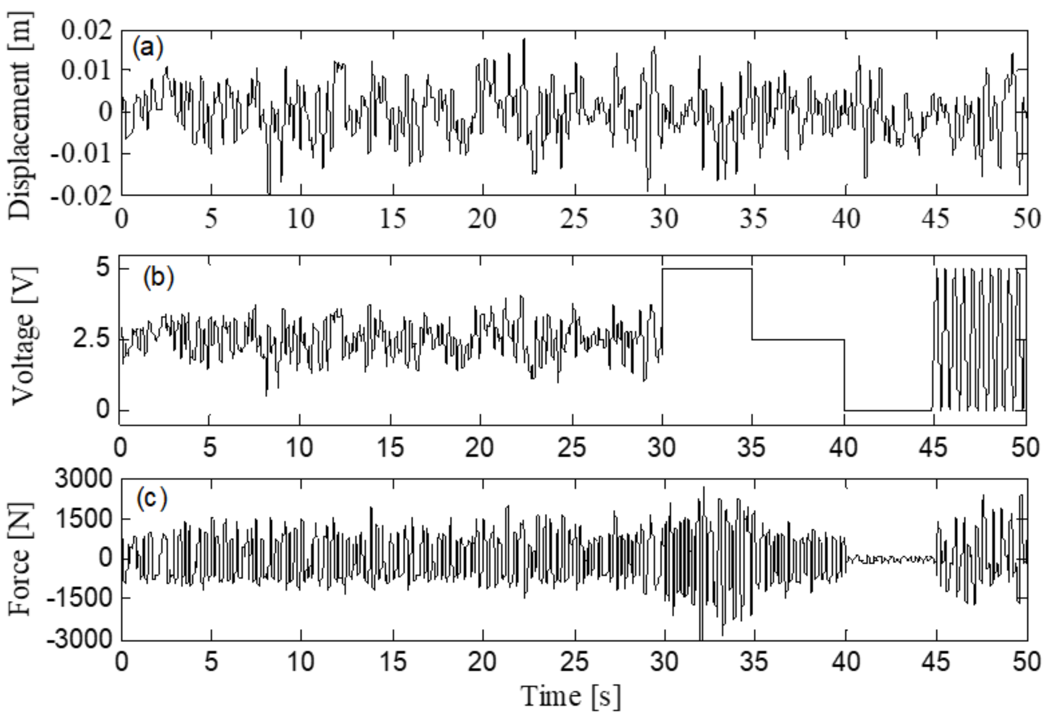


Figure 2: The time history of training data sets for neural network models: (a) displacement; (b) command voltage; (c) damping force

Neural Network Modeling of MR Fluid Dampers

Multilayer feedforward neural networks were shown to be capable of approximating any continuous function on a compact set conclusively in the late 1980s. So, in the last decade, neural networks have been proposed for the identification and management of nonlinear dynamical systems, in addition to addressing complicated issues in pattern recognition and time series prediction (Metered, Bonello, & Oyadiji, 2010). Figure shows the identification method for MR fluid dampers, in which the input u1(k) (composed of the displacement and command voltage) is linked to the MR fluid damper and the neural network model to be trained at the same time. Only the neural network model is coupled to u2(k). For the FNN model, the input u2(k) might be either the observed damping force and its delays or the anticipated damping force and its delays (for the RNN model). The difference e(k) between the output of the neural network model F (k) and the output of the MR fluid damper F(k) is used to modify the weights and biases of the neural network model until a 'sufficiently small' criterion is met. The training techniques and benefits of utilizing neural networks are determined by the design of the neural network model. The models of MR fluid dampers utilizing multilayer FNN and RNN models will be described in the next two subsections.

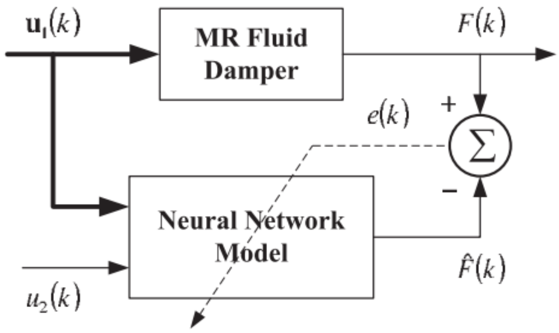


Figure 3: The scheme of identification for the MR fluid damper using the neural network model

Modeling of MR Fluid Dampers with FNN

The FNN may readily be used as the identification model for MR fluid dampers since it can approximate any continuous function (Liao & Wang, 2005). To comparisons, an FNN for modeling an MR fluid damper is also considered here. Figure 4 shows the scheme of the neural network, which represents the mapping

$$\begin{aligned}\hat{F}(k+1) = NN[&F(k), F(k-1), \cdot\cdot\cdot, F(k-IF+1),\\ &v(k), v(k-1), \cdot\cdot\cdot, v(k-Iv+1),\\ &x(k), x(k-1), \cdot\cdot\cdot, x(k-Ix+1)] \qquad (8)\end{aligned}$$

where NN [·] denotes a neural network with IF + Iv + Ix (=R) inputs and one output, trained to approximate the input–output mapping that describes the MR fluid damper. According to equation (8), the delays of F(k), v(k), and x(k) used as input to the neural network are IF −1, Iv−1, and Ix −1 respectively, denoted by blocks TDL (tapped delay line) in Figure 4. When the error e(k) between F(k) and ˆF (k) becomes sufficiently small, the NN [·] is well trained. For the identification of an MR fluid damper, a fully connected three-layer feedforward network with 18 (=S1) input layer neurons, 18 (=S2) hidden layer neurons, and 1 (=S3) output layer neuron.

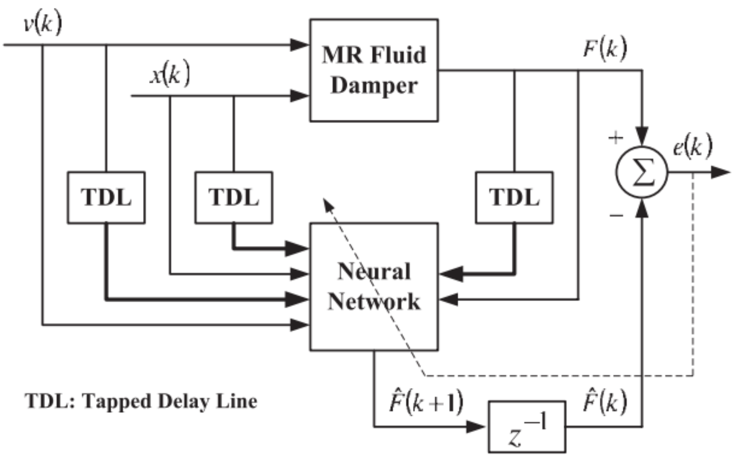


Figure 4: The scheme of identification for the MR fluid damper using the FNN model

Modeling of MR Fluid Dampers with RNN

Although the FNN model mentioned in the previous paragraph can reliably forecast the damping force of an MR fluid damper, the input and output information for the MR fluid damper must be examined during the neural network's training and prediction phases, which limits its use. It is desirable not to monitor the damping force using sensors when utilizing the neural network model to forecast the damping force online, which must be accomplished by putting one force sensor in series with each MR fluid damper. An RNN is used and trained in this part to estimate the force of the MR fluid damper. In this approach, the force in the MR fluid damper is only required during the neural network model's training stage. The force sensor is no longer required when the trained neural network model is used to forecast the damping force. Considering the foregoing, an RNN model is employed, in which the neural network model's output is delayed and sent back to its input layer. Figure 5 shows the scheme of the RNN model for an MR fluid damper and the mapping of the neural network is represented as

$$\hat{F}(k+1) = NN[\hat{F}(k), \hat{F}(k-1), \cdot\ \cdot\ \cdot, \hat{F}(k-OF+1), v(k), v(k-1), \cdot\ \cdot\ \cdot, v(k-Iv+1), x(k), x(k-1), \cdot\ \cdot\ \cdot, x(k-Ix+1)] \quad (9)$$

where NN [] is a neural network with Iv + Ix (=R) inputs and a single output that has been trained to approximate the input-output mapping that characterizes the MR fluid damper. The number of delays that the neural network's output sends back to the input layer is called OF.

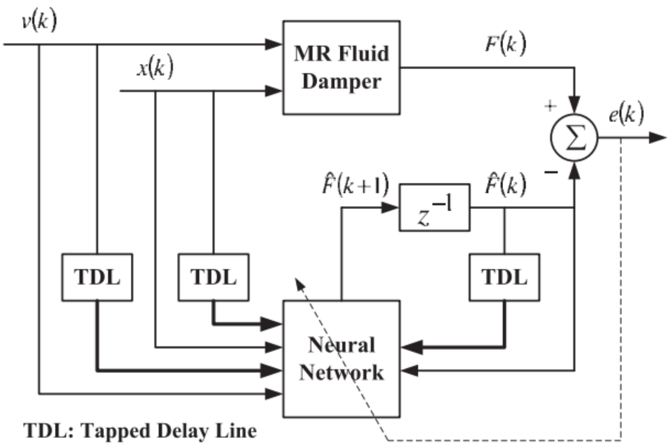


Figure 5: The scheme of identification for the MR fluid damper using the RNN model

Inverse Dynamic Modeling of MR Fluid Dampers

Inverse modelling with neural networks eliminates the need to explicitly invert the system's function. Because the damping force of an MR fluid damper is nonlinearly connected to the displacement across the damper and the command voltage, the inverse modelling of the MR fluid damper is divided into two scenarios:

i. The command voltage damping force when the neural network model's anticipated output u(k) Equals v(k) as illustrated in Figure 6(a);
ii. The damping force to displacement when the predicted output of the neural network model ˆ u(k) = ˆx(k) as shown in Figure 6(b).

Inverse modeling of the damping force to command voltage or displacement involves training a neural network model arranged in accordance with the configuration shown in Figure 6, in which u1(k) has the same meaning as defined in Figure 3. When the inverse relationship is modelled by the RNN, the predicted output ˆ u(k) from the RNN should be fed back to its input u2(k) (=ˆu(k)), which is denoted by the dashed line in Figure 6. As for the modeling with the FNN, u2(k) is the actual value that needs to be predicted by the FNN. In the case of modelling with the FNN, u2(k) is the actual value that the FNN must forecast. The neural network model approximates the inverse dynamics of the MR fluid damper by minimizing the error e(k) between the anticipated output u(k) of the neural network model and the target input u(k). When the damping force can be accessible, the results of the case (ii) can be utilized to estimate the displacement across the MR fluid damper, which is not the subject of this work. Case (i) findings may be utilized to regulate an MR fluid damper, which will be addressed in-depth in section 5.

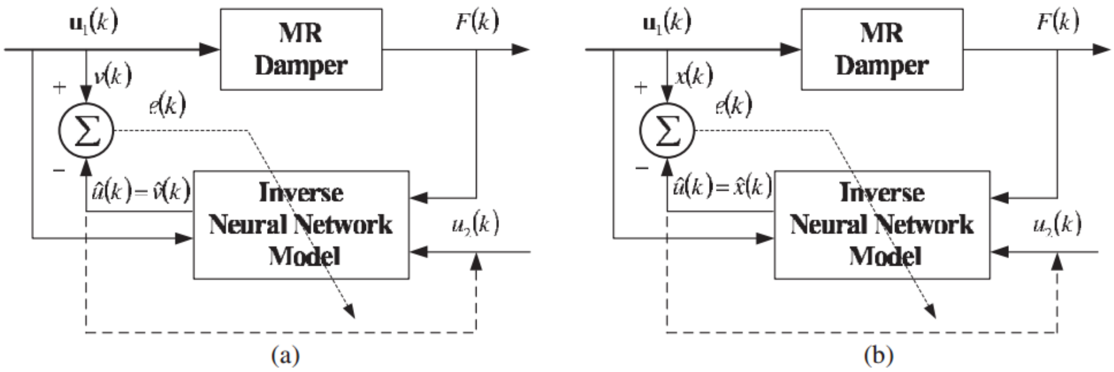


Figure 6: The scheme of the inverse modeling for the MR fluid damper using neural networks: (a) force to command voltage model; (b) force to displacement model

Modeling Inverse Dynamics of MR Fluid Dampers with the *FNN*

The neural network illustrated in Figure is trained to simulate the input-output behavior of an MR fluid damper when utilizing the FNN for inverse modeling. An FNN for simulating the inverse dynamics of an MR fluid damper is also utilized here for comparison; it is shown in Figure 7 for the mapping.

$$\hat{v}(k+1) = NN[v(k), v(k-1), \cdot\cdot\cdot, v(k-Iv+1), F(k), F(k-1), \cdot\cdot\cdot, F(k-IF+1), x(k), x(k-1), \cdot\cdot\cdot, x(k-Ix+1)] \quad (10)$$

where NN [] is a neural network with IF + Iv + Ix (=R) inputs and one output that has been trained to approximate the inverse input–output mapping that represents the inverse dynamic behaviour of the MR fluid damper.

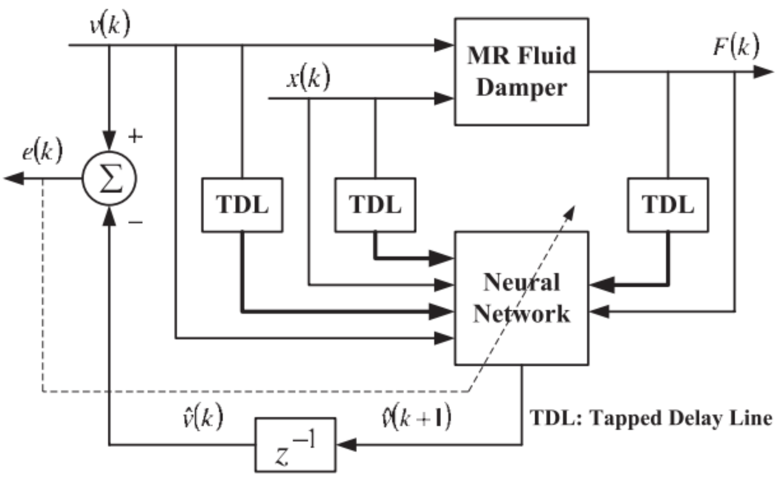


Figure 7: The scheme of the FNN for modeling the inverse dynamics of the MR fluid damper

Inverse Dynamic Modeling of MR Fluid Dampers

The validation scheme for the inverse modeling using FNN models for MR fluid dampers with SIMULINK is shown in Figure 8. In this figure, SIMULINK blocks labelled as MRD 1 and MRD 2 are created based on the modified Bouc–Wen model, which is used to represent the MR fluid damper in the validation process. The block labelled as MRD −NN−INVERSE represents the trained FNN model, which is to be validated. The displacement, the command voltage, and the damping force produced by MRD 1 are inputs to feed into the inverse FNN model to generate the command voltage signal, which is then fed into the MRD 2 together with the displacement to produce the predicted damping force. The validation process includes comparisons between the predicted command voltage and the input command voltage, the damping force predicted by MRD 2 and the target damping force by MRD 1. Only one validation case is presented here, as shown in Figure 10. The displacement and command voltage input are given by the validation set 5. Observing Figure 10, not only does the predicted command voltage coincide with the input command voltage, but also the damping force calculated using the predicted command voltage coincides with the damping force obtained with the input command voltage.

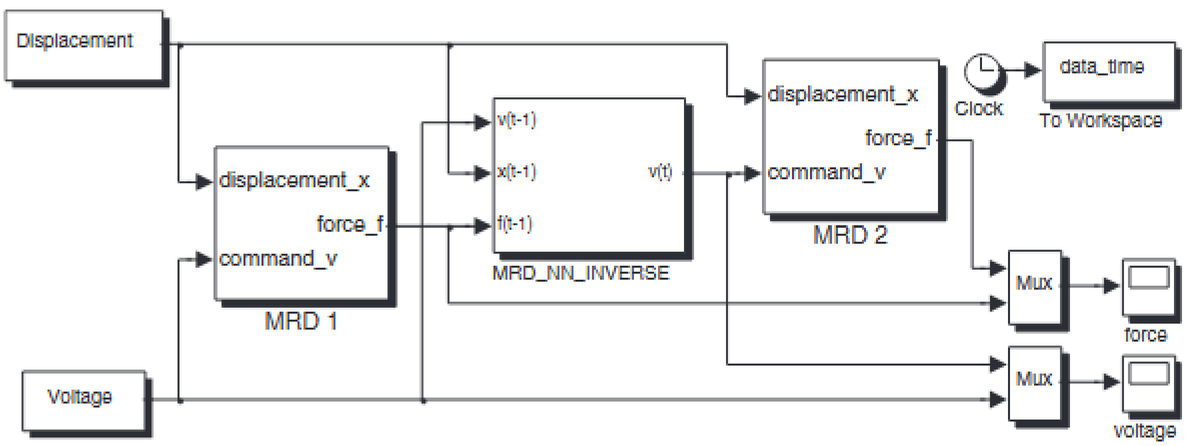


Figure 8: The validation scheme for the inverse modeling with the FNN model for the MR damper

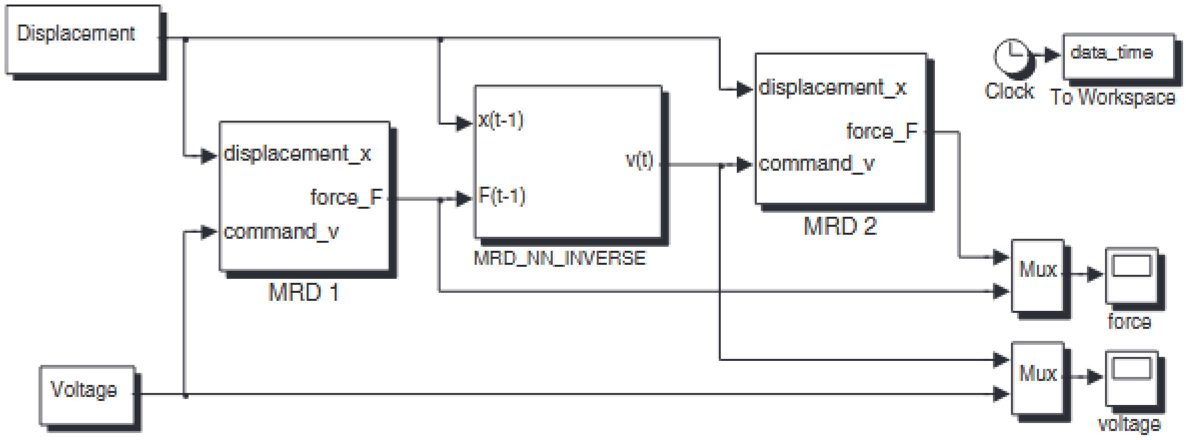


Figure 9: The validation scheme for the inverse modeling with the RNN model for the MR damper

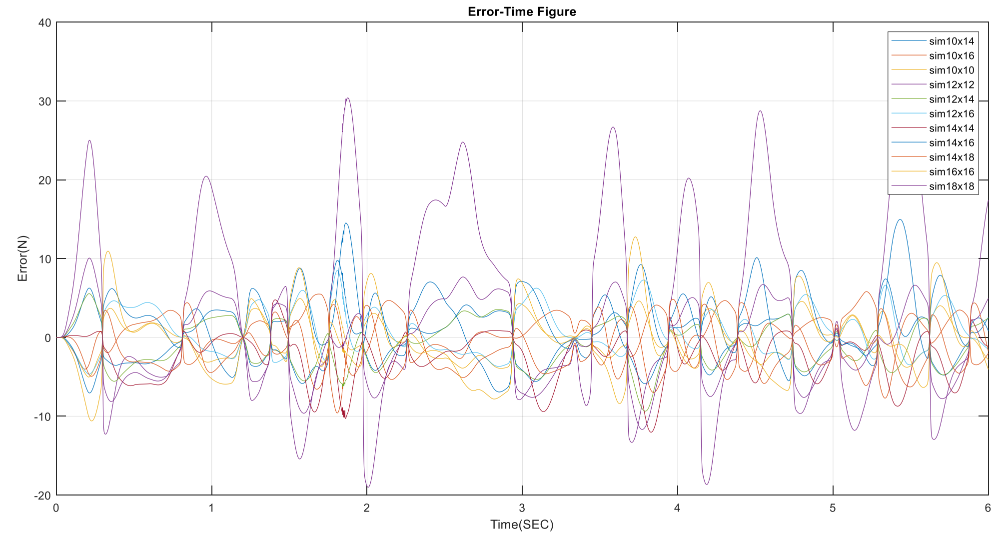
VALIDATION RESULTS AND DISCUSSION

To validate the neural networks introduced in the current paper, a series of validation data sets are listed and defined in Table 3. There are four validation cases are examined in this section for different 11 networks. The first validation set is demonstrated in Figure 10 for the direct FNN. The second one introduced in Figure 11 for the direct RNN. Figure 12 illustrates the results for the third validation set for the inverse FNN. The last validation set is shown in Figure 13 for the inverse RNN. From the above-mentioned validation results, the direct and inverse dynamic modeling for both FNN and RNN can predict the dynamic behavior of MR damper well. Moreover, the network with 14 neurons for the first hidden layer and 14 neurons for the second hidden layer offers the best tracking in the case of RNN and can use as a damper controller to reduce the response time of MR damper in semi-active suspension systems.

Table 3: Definition of validation data set

Validation Set	Displacement (cm)	Voltage (V)	Force (N)	Time span (sec)
1	$0.01\sin(4\pi t)$	1.5	Produced by modified Bouc-Wen model	6
2	$0.015\sin(2\pi t)$	$GWN^a + 2$		6
3	$0.02\sin(2\pi t)$	$2.5+2.5\sin(2\pi t)$		6
5	GWN^b	$2.5+2.5\sin(2\pi t)$		6

Note: a – Gaussian white noise (frequency: 0-2 Hz; amplitude: ± 2); b – Gaussian white noise (frequency: 0-2 Hz; amplitude: ± 2).



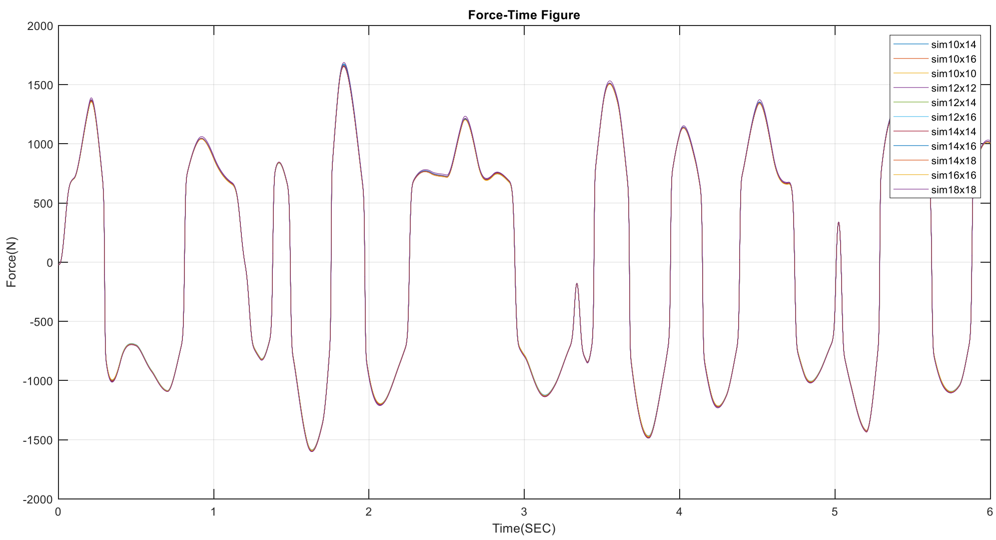
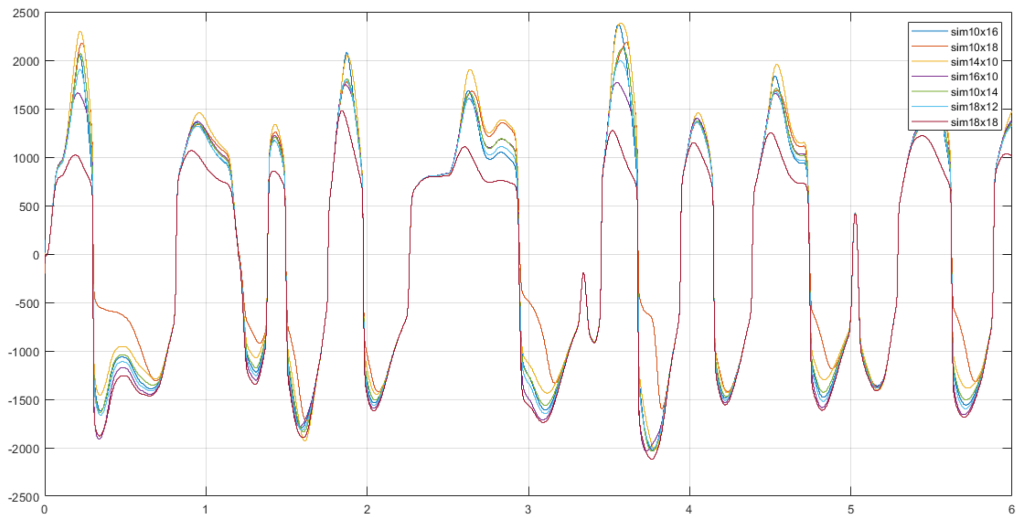


Figure 10: Direct FNN prediction



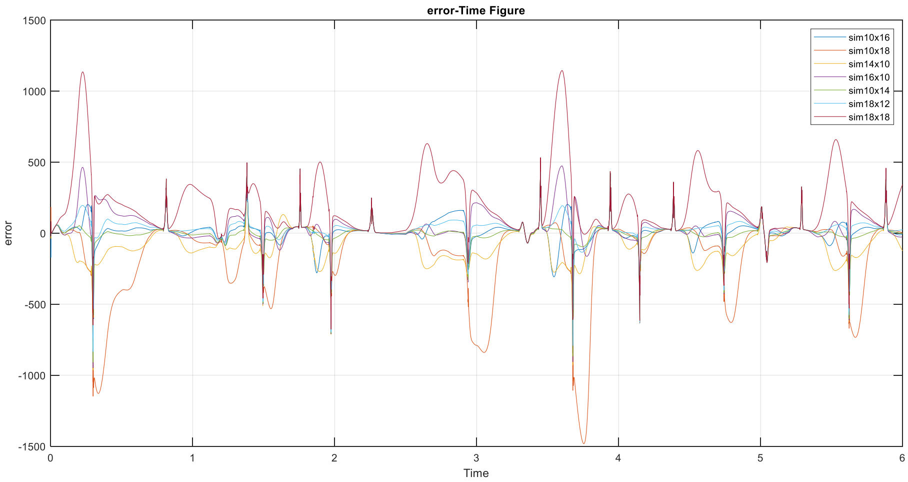
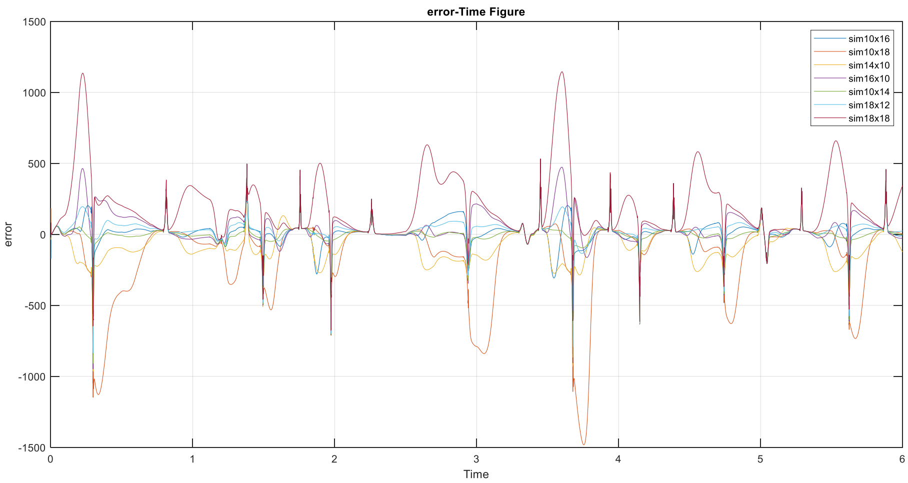


Figure 11: Direct RNN prediction



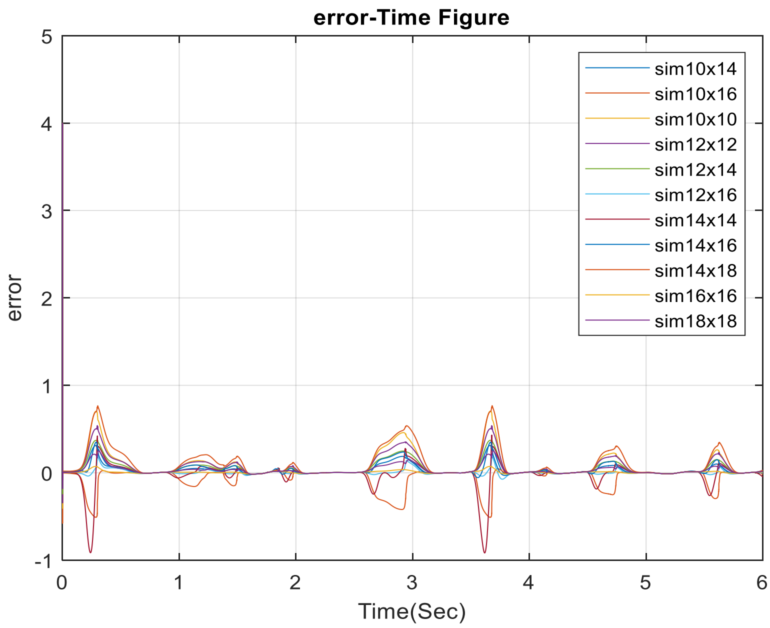
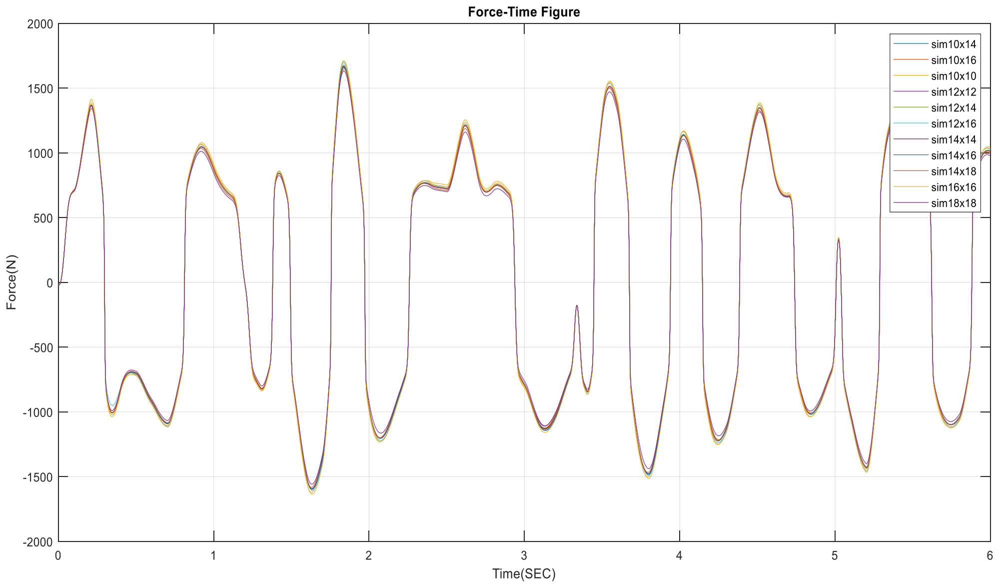


Figure 12: Inverse FNN prediction



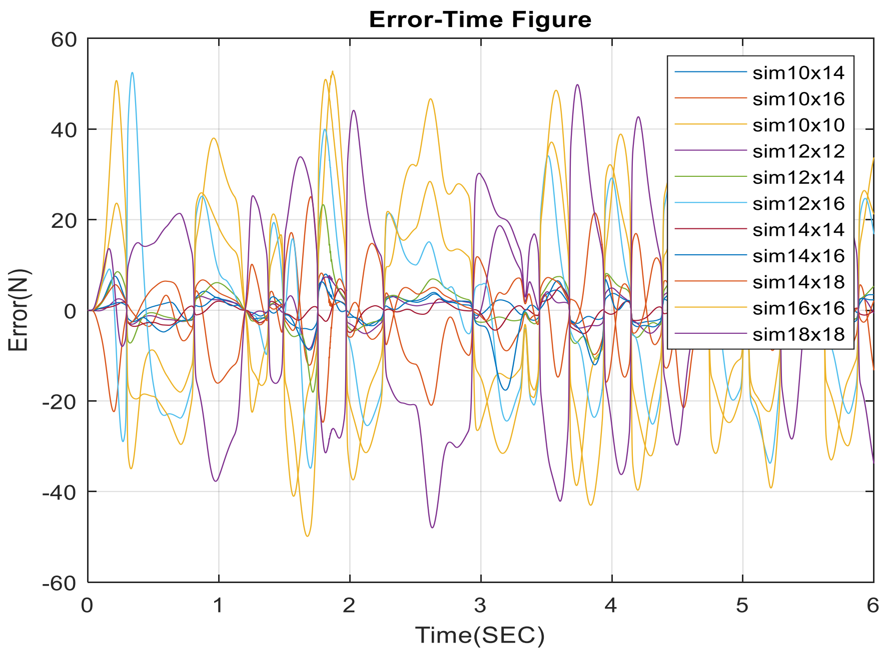


Figure 13: Inverse RNN prediction

CONCLUSIONS

In this paper, the usage of neural network (NN) is studied to identify the forward and inverse nonlinear dynamic behavior of magnetorheological (MR) fluid dampers. The direct NN model can predict the MR damper force based on the displacement and input voltage to the damper coil. The inverse NN model can generate the input voltage based the displacement and MR damper force. The details and training techniques for the direct and inverse NN models for the MR damper are introduced. Both NN models for direct and inverse are trained and validated using the input and output data of the well-known Bouc-Wen mathematical model. Theoretical validation results reveal that the proposed direct and inverse NN models can predict the nonlinear dynamic behavior of MR dampers precisely.

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