EJSIT
European Journal of Science, Innovation and Technology
ISSN: 2786-4936
www.ejsit-journal.com
Volume 4 | Number 1 | 2024

A Comparison of the Existing Ellipsoidal to Orthometric Height Transformation Models

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ABSTRACT

Geodetic height transformations play a crucial role in bridging the gap between ellipsoidal and orthometric height systems, facilitating accurate elevation data integration across diverse applications. This study presents a comprehensive comparison of existing ellipsoidal to orthometric height transformation models, aiming to assess their performance and identify potential areas for improvement. This paper focuses on evaluating the accuracy, reliability, and computational efficiency of prominent transformation models, including but not limited to Molodensky, Helmert, and rigorous mathematical approaches. Various datasets, encompassing different geographic regions and terrain characteristics, are employed to ensure a comprehensive assessment of model performance under diverse conditions. Key metrics such as root mean square error (RMSE), bias, and standard deviation are utilized to quantify the differences between transformed orthometric heights and observed values. Additionally, computational aspects such as algorithm complexity and processing time are considered to evaluate the practicality of each model for large-scale applications. The findings of this study contribute to the understanding of the strengths and limitations of existing ellipsoidal to orthometric height transformation models, providing valuable insights for geodetic practitioners, surveyors, and researchers. The identified areas for improvement may guide future developments in height transformation methodologies, advancing the accuracy and reliability of elevation data integration in geospatial sciences. Ultimately, this research aims to enhance the precision of height information crucial for applications ranging from engineering and construction to environmental monitoring and geospatial analysis.

Keywords: GNSS, ellipsoidal and orthometric height, ITRF/ETRF/ALB reference, transformation

INTRODUCTION

In the realm of geodetic sciences, the accurate representation and transformation of elevation data are pivotal for a myriad of applications spanning from civil engineering projects to environmental monitoring and spatial analysis. The geodetic community faces the constant challenge of harmonizing data acquired in different reference frames, particularly when dealing with ellipsoidal and orthometric height systems. This challenge necessitates the development and refinement of height transformation models, aiming to bridge the gap between these two fundamental representations of elevation (Featherstone et al., 1998).

The transition from ellipsoidal to orthometric heights involves correcting for the irregularities in the Earth's gravitational field, introducing complexities that demand sophisticated mathematical formulations and transformation methodologies. Numerous models have been proposed over the years, each offering a unique approach to address the intricacies inherent in height transformations. These models range from classical techniques like the Molodensky and Helmert transformations to more advanced mathematical formulations designed to account for regional variations in the Earth's gravity field.

This study undertakes a comprehensive exploration and comparison of the existing ellipsoidal to orthometric height transformation models, acknowledging the importance of a nuanced evaluation in selecting the most appropriate method for specific applications. As the demand for precise geodetic information continues to grow in fields such as navigation, land surveying, and environmental monitoring, the need for reliable height transformation models becomes increasingly pronounced.

The motivation behind this comparative analysis lies in the imperative to scrutinize the strengths and weaknesses of various transformation models under diverse conditions. Addressing this issue is crucial not only for ensuring the accuracy of elevation data but also for guiding practitioners, surveyors, and researchers in choosing the most suitable model for their specific needs. By providing a nuanced understanding of the performance of existing models, this study aims to contribute valuable insights that can inform both the selection and further development of ellipsoidal to orthometric height transformation methodologies (Al-Kadi, 1989).

Throughout this exploration, the research will delve into the intricacies of commonly employed models, assess their accuracy using real-world datasets from diverse geographic regions and terrains, and scrutinize computational aspects such as algorithmic efficiency. The overarching goal is to not only evaluate the status quo but also to identify potential areas for enhancement, ultimately advancing the precision and reliability of height transformations in the evolving landscape of geodetic sciences. This comparative analysis thus serves as a critical step towards refining geodetic methodologies and ensuring the seamless integration of elevation data in an increasingly interconnected and technology-driven world.

During Autumn 2007 - Spring 2008, the GNSS measurement campaign is carried out by IGUS (Military Geographical Institute of Albania) and the Military Geographical Institute of Florence (IGUF), Italy. The aim was to find the transformation parameters from the European reference system (ETRF-2000, Epoch 2008) to local (ALB-1986). GNSS surveys were carried out at 150 points of state geodetic network (Figure 1). The coordinates of the points are calculated in ITRF-2005 and transformed into ETRF-2000, Epoch 2008.0 (Nurce, 2013).

Figure 1: Scheme of the points measured Autumn 2007 - Spring 2008

Based on 125 common points, 7-Parameters of Helmert transformation are calculated in advance (Table 1) with "start" system ETRF2000, Epoch2008 (with height hGRS80 and H EGM2008) and "arrival" system again ETRF2000, Epoch 2008 with true geoid heights (ALB86 leveling) (Maseroli, 2010).

Table 1: 7P Helmert for height transformation of hGRS80, EGM2008 into HALB87

T_X (m)	T_Y (m)	T_Z (m)	S (ppm)	R_X ('')	R_Y ('')	R_Z ('')
5.792	-13.506	-0.902	0.0611	0.2800	0.1440	-0.3581

From the ellipsoidal zone (between the parallels 39°÷43°.5 and the meridians 18°÷22°) adaptation of the above 7-parameters of the Helmert transformation with EGM2008 model, the geoidal waves N for the Albanian territory have been calculated. N of any point (not grid vertices) was calculated through a bilinear interpolation of the waves (z) within the 4 vertices of the grid cell with dimensions 2'.5x2'.5 (Figure 2), with a polynomial of the form (Maseroli, 2010):

$$z = (1-s)\,(1-t)\,z_{00} + t\,(1-s)\,z_{01} + s\,(1-t)\,z_{10} + s\,t\,z_{00}$$

where:

$$x - x_0/x_1 - x_0 = s, \qquad y - y_0/y_1 - y_0 = t$$

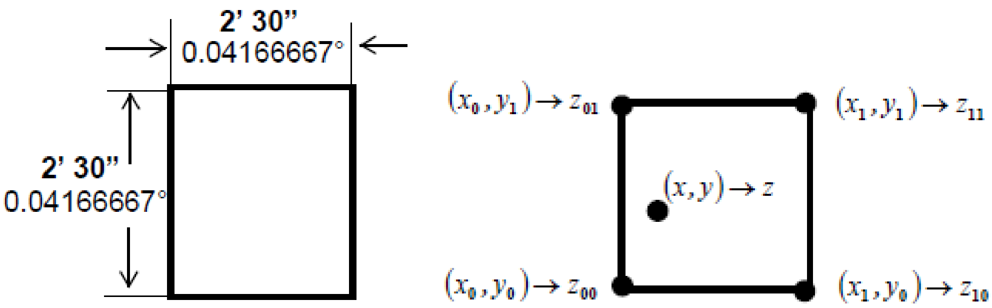


Figure 2: Grid cell dimensions

The geoid height (H_{ALB86}) is calculated from the respective ellipsoidal height (h_{GRS80}) according to the relation:

$$H_{ALB86} = h_{GRS80} + N_{EGM2008\text{ - }GRS80}$$

Errors of the transformation of ellipsoidal heights h_{GRS80} into orthometric $H_{ALB1987}$ through ***ALBGEO3*** model are estimated: ±20 cm at confidence level of 68% and ±40 cm at the confidence level of 85%.

METHODOLOGY

Based on 3-D coordinates of the common points of the GNSS measurement campaign Autumn 2007 - Spring 2008 and the so-called orthometric heights H, for the transformation of the ellipsoidal heights h referring GRS80 into the orthometric height H referred to ALB1987 ***a linear interpolation polynomial of degree 5 with three variables (x, y, z)*** was proposed by the Department of Geodesy, Faculty of Civil Engineering, of the form:

$$P_5^3(x,y,z) = P_4^3(x,y,z) + a_{35}x^4y + a_{36}x^4z + a_{37}x^3yz + a_{38}x^3z^2 + a_{39}x^2y^2z + a_{40}x^2yz^2 + a_{41}x^5 + \\ + a_{42}y^4x + a_{43}y^4z + a_{44}y^3xz + a_{45}y^2xz^2 + a_{46}y^5 + a_{47}z^4x + a_{48}z^4y + a_{49}z^3xy + a_{50}z^5 + a_{51}x^3y^2 + \\ + a_{52}y^3x^2 + a_{53}y^3z^2 + + a_{54}z^3x^2 + a_{55}z^3y^2$$

or in matrix form:

$$A_{n,\,55} \cdot X_{55,\,1} = L_{n,\,1}$$

where, $x = d\varphi_i = \varphi_i - \varphi_0$, $y = d\lambda_i = \lambda_i - \lambda_0$, $z = dh_i = h_i - h_0$ are the corresponding differences between the latitudes, longitudes, and ellipsoidal heights of the points in common in both systems (Maseroli, 2010).

The transformed height in the ALB1987 is given as:

$$H_{ALB1987} = h_{GRS80} + P_5^3(x, y, z)$$

Once the barricenters have been calculated (φ_0, λ_0, h_0):

$$\varphi = \Sigma\varphi_i/n = 0.717224124^{\text{ rad}},\ \lambda_0 = \Sigma\lambda_i/n = 0.350605742^{\text{ rad}},\ h_0 = \Sigma h_i/n = 637.7247\text{ m}$$

and the polynomial coefficients are found: a_1, a_2, a_3, ..., a_{54}, a_{55}.

Average quadratic deviation in the transformation of ellipsoidal heights h referred to GRS80 into orthometric heights H referred to ALB1987 has resulted: $\sigma_0 = \pm 0.1567$ m.

Errors of the transformation of ellipsoidal heights h (GRS80) into orthometric H (ALB1987) through the polynomial model found by the Department of Geodesy, Faculty of Civil Engineering were estimated (Nurce, 2013):

0÷10 cm 55% of cases, ÷20 cm 80%, ÷30 cm 92%, ÷40 cm 98%, >40 cm 2%.

Field Testing of the Transformation Models

To test the transformation models of ellipsoidal to geoidal heights, static GNSS satellite measurements were performed (Figure 3), at the points referred ALB1986, which were not used to derive the transformation models in plan and height during Autumn 2007- Spring 2008 GNSS campaign.

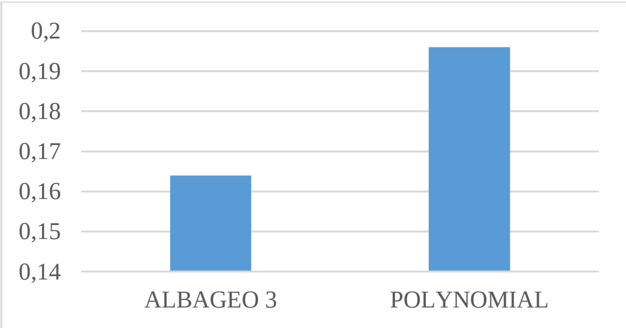
Figure 3: Test areas of transformation models of h to H

After processing of GNSS observations with Trimble Business Center software we have the list of points with 3-D coordinates (φ, λ, h) and (N, E, h) referred ETRF-2000, Epoch 2022.5.

The Tables 2, 3, …., 6 given the transformed heights from reference ETRF2000, Epoch2022.5 into reference ALB1987 through 2 transformation models: (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5, as well as the corresponding quadratic mean deviations σ_{01} and σ_{02}.

Table 2: Transformed heights hGRS80 → HALB1987 for the measured points in the area 1

No.	$H_{ALB1987}$ *(m)*	$h_{GRS80} \rightarrow H_{ALB1987}$ (LIP of degree 5)	v_1 (m)	$h_{GRS80} \rightarrow H_{ALB1987}$ (ALBGEO3)	v_2 (m)
1	361.036	360.946	0.090	361.189	-0.153
2	316.76	316.666	0.094	316.931	-0.171
3	……………	……………	……………	……………	……………
4	……………	……………	……………	……………	……………
17	240.476	240.59	-0.114	240.770	-0.294
			σ_{01} = 0.164 m		σ_{02} = 0.196 m



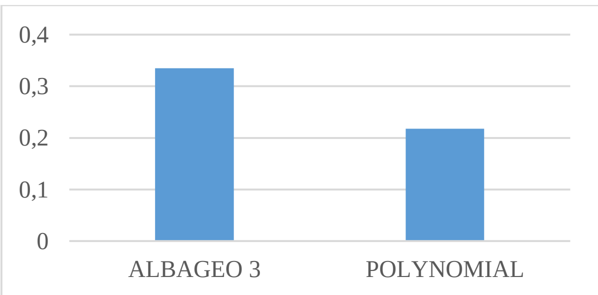
Errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of area 1 through (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5 are respectively:

(1) 0÷10.0cm 41% of cases, ÷20.0cm 76.5%, ÷30.0cm 100%,

(2) 0÷10.0cm 18% of cases, ÷20.0cm 53%, ÷30.0cm 82.5%, ÷40.0cm 100%.

Table 3: Transformed heights for the measured points in the area 2

No.	$H_{ALB1987}$ *(m)*	$h_{GRS80} \rightarrow H_{ALB1987}$ (LIP of degree 5)	v_1 (m)	$h_{GRS80} \rightarrow H_{ALB1987}$ (ALBGEO3)	v_2 (m)
1	603.997	604.026	-0.029	604.064	-0.067
2	577.522	577.33	0.192	577.332	0.190
3	……………	……………	……………	……………	……………
4	……………	……………	……………	……………	……………
8	810.035	810.123	-0.088	810.094	-0.059
			σ_{01} = 0.335 m		σ_{02} = 0.218 m

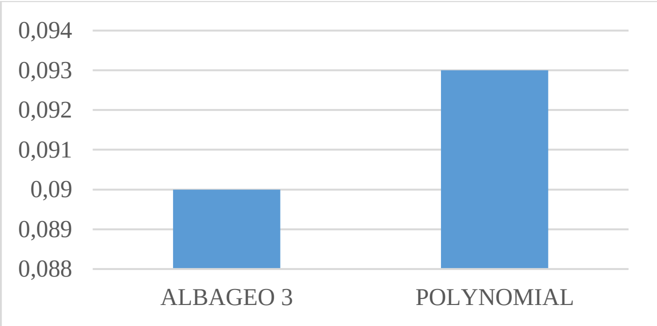


Errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of area 2 through (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5 are respectively:

(1) 0÷10.0cm 37.5% of cases, ÷20.0cm 100%,
(2) 0÷10.0cm 50% of cases, ÷20.0cm 87.5%, >40.0cm 100%.

Table 4: Transformed heights hGRS80 → HALB1987 for the measured points in the area 3

No.	$H_{ALB1987}$ *(m)*	$h_{GRS80} \rightarrow H_{ALB1987}$ (LIP of degree 5)	v_1 (m)	$h_{GRS80} \rightarrow H_{ALB1987}$ (ALBGEO3)	v_2 (m)
1	34.95	34.81	0.140	34.877	0.073
2	37.93	37.89	0.040	37.964	-0.034
3	……………	……………	……………	……………	……………
4	……………	……………	……………	……………	……………
6	370.474	370.434	0.040	370.317	0.157
			σ_{01} = 0.090 m		σ_{02} = 0.093 m

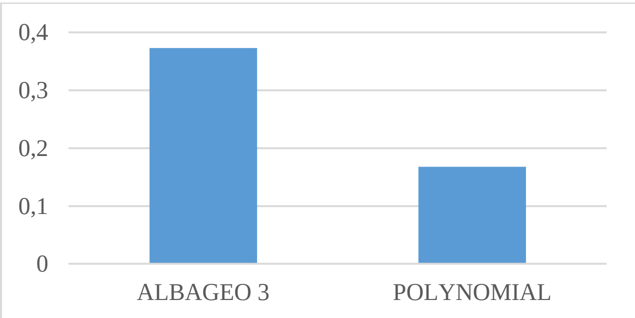


Errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of area 3 through (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5 are respectively:

(1) 0÷10.0cm 50% of cases, ÷20.0cm 66.7%, >40.0cm 100%,
(2) 0÷10.0cm 50% of cases, ÷20.0cm 66.7%, >40.0cm 100%.

Table 5: Transformed heights hGRS80 → HALB1987 for the measured points in the area 4

No.	$H_{ALB1987}$ *(m)*	$h_{GRS80} \rightarrow H_{ALB1987}$ (LIP of degree 5)	v_1 (m)	$h_{GRS80} \rightarrow H_{ALB1987}$ (ALBGEO3)	v_2 (m)
1	141.201	140.698	0.503	140.921	0.280
2	179.152	178.693	0.459	179.059	0.093
3	……………	……………	……………	……………	……………
4	……………	……………	……………	……………	……………
10	829.805	829.856	-0.051	829.610	0.195
			σ_{01} = 0.373 m		σ_{02} =0.168 m



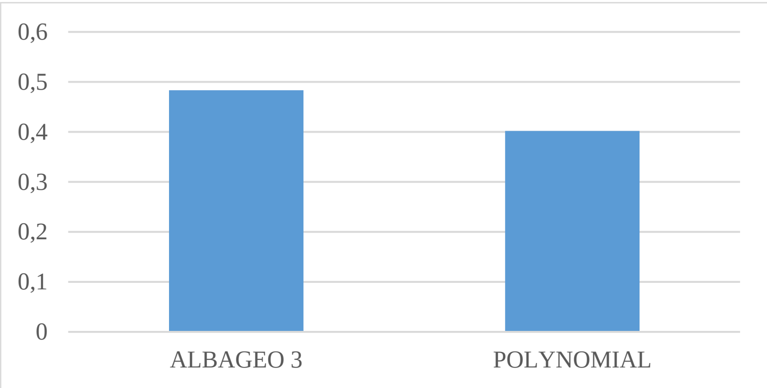
Errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of area 4 through (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5 are respectively:

(1) 0÷10.0cm 20% of cases, ÷20.0cm 30%, ÷30.0cm 40%, ÷40.0cm 50%, >40.0cm 100%,

(2) 0÷10.0cm 50% of cases, ÷20.0cm 80%, ÷30.0cm 90%, >40.0cm 100%.

Table 6: Transformed heights hGRS80 → HALB1987 for the measured points in the area 5

No.	$H_{ALB1987}$ *(m)*	$h_{GRS80} \rightarrow H_{ALB1987}$ (LIP of degree 5)	v_1	$h_{GRS80} \rightarrow H_{ALB1987}$ (ALBGEO3)	v_2
1	826.900	826.5796	0.320	826.632	0.268
2	777.341	776.8248	0.516	776.832	0.509
3	……………	……………	……………	……………	……………
4	……………	……………	……………	……………	……………
23	566.881	566.7907	0.090	566.945	-0.064
			σ_{01} = 0.483m		σ_{02} =0.402m



Errors of the transformation of the ellipsoidal heights h (GRS80) into orthometric H (ALB1987) at the measured points of area 5 through (1) ALBGEO3 software and (2) linear interpolation polynomial (LIP) of degree 5 are respectively:

(1) 0÷10 cm 26% of cases, ÷ 20 cm 61%, ÷ 30 cm 92%, ÷ 40 cm 92%, > 40 cm 8%,

(2) 0÷ 10.0 cm 35% of cases, ÷ 20 cm 52%, ÷ 30 cm 57%, ÷ 40 cm 74%, > 40 cm 26%.

For all points (64 points): σ_{01}= 0.231 m, σ_{02}=0.165 m.

CONCLUSION AND RECOMMENDATIONS

1. The Linear Interpolation Polynom of 5th degree proposed by B. Nurçe (2013) compared to the ALBGEO-3 model developed by MGIF has the following differences and advantages:

- no needs knowledge about the EGM-08.
- achieves higher accuracy of the height transformation compared to the ALBGEO-3.

2. Both models of transformation of h referred GRS80 to H referred ALB1987, are not officially approved, recommended to be used for the topographic maps up to 1: 5000.

3. Both models of the transformation of the ellipsoidal heights (h) referred GRS80 to the orthometric heights (H) referred ALB-1987 can be improved if we will perform additional measurements in existing points of the vertical control network referred ALB-1987.

REFERENCES

Maseroli, R. (2010). Relacione sui lavori geodetici eseguiti sul territorio Albanese realizzazione della rete geodetica satellitare.

Nurce, B. (2013). The development of H and V references of Albania.

Featherstone, W. E., Dentith, M. C., & Kirby J. F. (1998). Strategies for the accurate determination of orthometric heights from GPS. *Survey Review*, *34*(267), 278-296.

Al-Kadi, I. (1989). GPS satellite surveying in developing countries- a review. *Eng. Sci, 1*(1-2), 95-109.

Konakoglu, B., Cakir, L., & Gockalp, E. (2016). 2D coordinate transformation using artificial neutral networks. *Int. Arch. Photogrammetry*, *Remote Sens. Spatial Inf. Sci – ISPRS Archives*, *42*, 183-186.