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Production System Optimisation with a Multi-Level Process Control Tool Integrated into Maintenance and Quality Strategies under Services and Quality Constraints

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ABSTRACT

This study investigated an optimal production, maintenance, and quality forecasting problem, with an improved statistical process chart of a supply chain under service and quality requirements. The product quality control is done using a multi-level statistical process chart with additional surveillance. The tool indicates when the process is respectively in control, on surveillance, and under critical stages. While the process is in a control state, producing conforming items is subjected only to minimal corrective maintenance. But when the operation falls to a surveillance stage it requires imperfect preventive maintenance with duration to reduce the failure rate. And when the operation moved to a critical state, then a perfect maintenance action with significant time is employed to restore the process to (AGAN). The decision variables are the sample size, the control interval, the control charts' surveillance limits and the critical limits. The multi-ware houses are to satisfy random customer demands during the finite production horizon with service, quality, and production bounds constraints. We use a stochastic mathematical formulation, simulation, and optimization to determine the optimal chart parameters, which minimizes the total expected cost of production. To highlight the interesting aspects of the interactions between production, quality, and maintenance, numerical examples and sensitivity analysis are presented. The work reduces the non-conformal losses and, subsequently, achieved total cost minimization.

Keywords: optimisation, production, maintenance, quality, control chart, cost

INTRODUCTION

There is a need for industrial companies to reduce their costs, satisfy multiple customer requirements, and at the same time make profits while maintaining market competitiveness. Production, maintenance, and quality are the most critical aspects of an industrial system, and these factors are interrelated with each other (Abubakar et al., 2022). Production planning reduces the work in progress and ensures the satisfaction of the demand (Chen et al., 2015). Maintenance increases availability with a reduction of failures at increased reliability (Enderlein, 1966). While quality control will guarantee the desired product quality (Aarab et al., 2017). Therefore, the integration of these fundamental interdependent factors will result in achieving improved production system efficiency as well as significant total production cost minimization (Bahria et al., 2021). However, the planning and implementation of these factors jointly and optimally represent a challenge to production companies (Xiao et al., 2019). And traditionally, these factors were not treated altogether or optimally (Farahani et al., 2019). But, with the development of the current industrial system, the emergence of new products, competitive markets, required the collaboration of these interrelated factors to achieve overall objectives (Abubakar et al., n.d.). According to Abubakar et al. (2020), customers are becoming more exigent, while production systems are getting more complex and subject to multiple uncertainties, also, demand becomes random. Buzacott (1967) is one of the earliest authors to accomplish the improvement of production. In his work "Automatic transfer lines with buffer stock," he studied the role of buffer stocks in boosting system productivity to solve the challenge of integrating maintenance into production plans. Beheshti Fakher et al. (2017) studied a multi-period multi-product capacitated lot-sizing context, integrating production, maintenance, and quality for an imperfect process. Guo et al. (2022) consider lot sizing, quality, and maintenance for an imperfect production system. Rivera-Gómez et al. (2021) proposed an integrated production maintenance and quality control policy for an unreliable manufacturing system subject to degradation. A minimal repair is carried out at failure to restore the production system to its previous status (ABAO). Darendeliler et al. (2020) investigate production, preventive maintenance, and quality control using a sampling method. The production lot size, the sampling plan, the safety stock, and overhaul planning are under quality constraints. The work of Hadian et al. (2021) determined the optimal buffer stock size of the production system subjected to periodic preventive maintenance (PM) action, thereby reducing the probability of failure and the machine age, proportional to the preventative maintenance. Nahas (2017) created a stochastic model of unstable equipment, which focused on joint optimization of economic production and aged-based preventive maintenance policy for deteriorating production systems using proportional hazard (PHM). Recently, production and quality issues were attended to by Hajej et al. (2021) in their work, which integrated the model of production, maintenance, and quality. The work studied a randomly failing manufacturing system that has to satisfy random demand during a finite production horizon and under a given service level. Other researchers who dealt with maintenance and quality are Ait-El-Cadi et al. (2021), who worked on integrated optimization of Production planning, maintenance, and quality control policy without considering the effects of inventory control shortages.

To guarantee the continuity and stability of the process reliability, several works proposed an optimal maintenance strategy related to the control chart. Ben-Daya and Rahim (2000) study the effect of maintenance on the economic design of the x-control chart, and its expansion. Si et al. (2018) suggested a reliability and maintenance structure for a two-state process optimizing decision to identify discrete timeframes for preventive maintenance tasks. Nevertheless, they also neglected inventory scarcity. Salmasnia et al. (2017) use a particle mass optimization algorithm, to achieve a collaborative design of production run length, maintenance policy, and control chart is built using numerous assignable reasons. We can cite few works who uses control charts, but make assumptions that the machine degradation is constant. Considering these major limitations, we propose in this work to have both preventive and corrective maintenance actions triggered by the control chart. Also, to deal with a large number of common cases, small drifts as well as some special causes of process variation, we establish additional surveillance limits. The surveillance limits monitors and triggers imperfect preventive maintenance actions to reduce the increasing failure rates. Depending on the control chart measurement results, one can decide whether to undertake or not the type of maintenance (preventive or corrective) actions at the end of each sampling interval following the production and the degradation of the process. This reinforces the integration of maintenance and quality control compared to existing strategies in the literature for which PM actions are not triggered directly by the process control. Moreover, the proposed approach will also simultaneously take into account production through the degradation degree of the process in the maintenance.

The originality of this work is in its collaborative determination of production and inventory quantities under varying customer demands and throughout the finite production horizon. A mathematical model is developed to determine the optimal values of the sample size, the sampling interval, the surveillance, and the control limits of the control chart, which minimize the production, inventory, non-conformal products, maintenance, and quality costs. The remainder of the paper is organized as follows: Section 2 described the problem of the study, its motivation, and targeted contributions. Section 3 present the stochastic model of production, maintenance, and quality problem. Section 4 deals with the sequential optimisation procedure employed in this model. Section 5 reported the numerical experiment conducted, the results and its discussion, as well as the sensitivity analysis, are presented. Section 6 presented the drawn conclusion of the study.

PROBLEM DESCRIPTION AND NOTATIONS

Targeted Contributions

Although the normal control chart is widely used and regarded as an effective tool in statistical quality control. In today's world, the pursuit of high-quality products is asking for high precision quality control in the present and future industrial applications. Most of the works, we reviewed, which used the normal control charts for the detection of a process variation suggested the need for improvement. This work presented recent development in the design and application of an improved statistical Process control chart (ISPC) with additional control limits as decision variables. Figure 1 below illustrates the improved process chart.

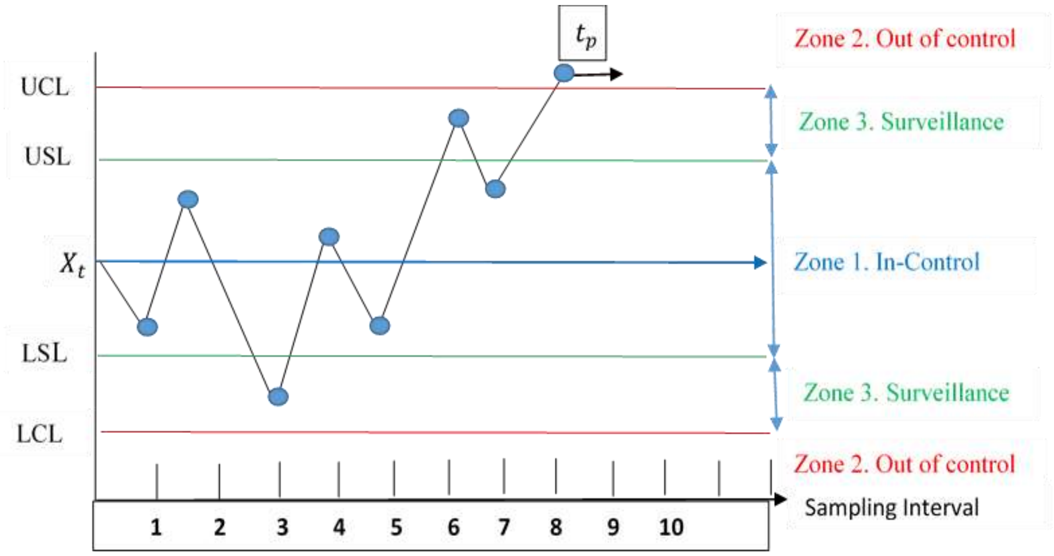

Figure 1. Control chart with surveillance

Production Problem Description

The production system composes of a single machine that produces only one sort of product, linked to a supply chain. The supply chain is made up of a principal manufacturing store$\mathbf{S}$, connected to multi-purchase warehouses $(\mathbf{w_1},\ \mathbf{w_2}, \ldots\ \mathbf{w_L})$ with a delivery volume capacity $\mathbf{Q_v}$ and time when the customers receive their demand products. The system operates at a given service level $\boldsymbol{\theta_i}$, and production capacity $(\mathbf{U_{min}} \leq \mathbf{u(k)} \leq \mathbf{U_{max}})$ over a finite horizon $\mathbf{H}$. The production system is unreliable, hence subject to faults and repairs at any time. The deterioration of the machine is influenced by the use and production cadences, also, the control chart variables. Consequently, the failure rate $\boldsymbol{\lambda(t)}$ increases with time, and the production rate $\mathbf{u(k)}$ affecting the reliability and of the production process responsible for non-conforming products. Figure 2 below represent the proposed model.

Principles and Characteristics of the Control Chart

Figure 2 above illustrates the schematic usage of the control chart in a production line. The ISPC is a tool with additional limits to monitor processes in multiple process scenarios, hence can guide the administration of multiple maintenance actions. This will therefore guarantee a more precision requirement of a high-quality product. Consider Figure 3 below, if the sample means are located within the control limits, before drifting to the surveillance limits the process is in control, where corrective maintenance with minimal repairs is employed. And if the mean value is found outside the control limits, then the process is in critical state, hence the machine is stopped and a perfect maintenance action is employed with time $(\boldsymbol{t_p})$. But if the process is found in between the in-control and the out-of-control states is said to be within the surveillance limits, in this case, imperfect maintenance is carried out. The ISPC are designed to achieve high-precision products. Also to alert small drifts or ascertain some uncertainties in-between the target and off-target.

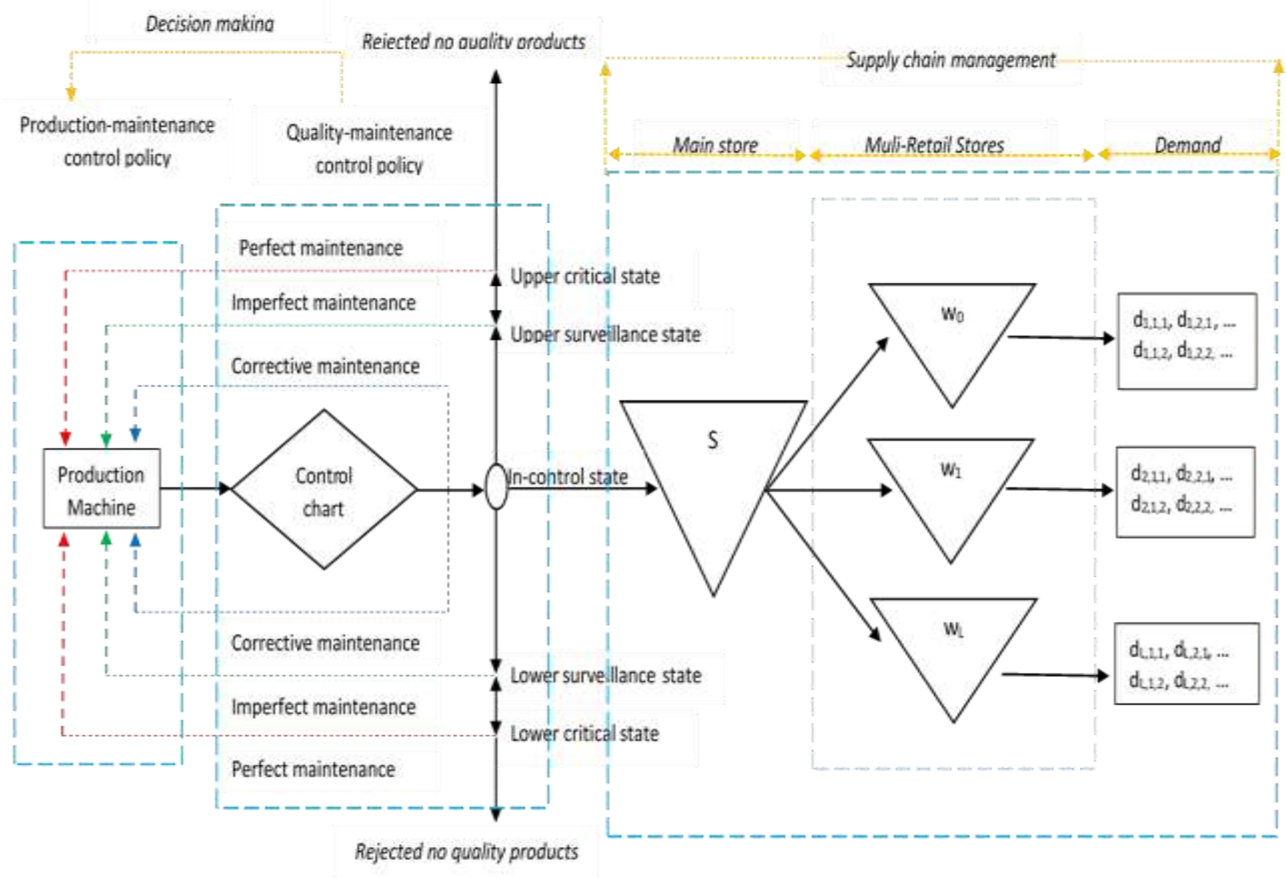

Figure 2. Production Maintenance and Quality System link to the supply chain

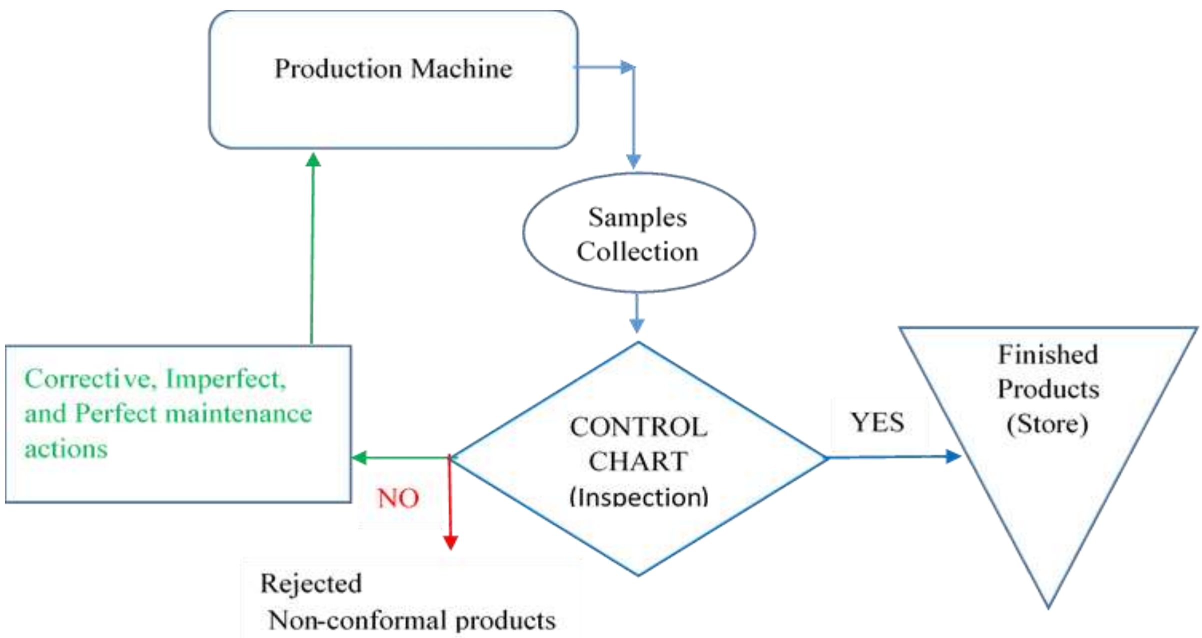

Figure 3. Control Chart Monitoring of Production

Production Process Scenario with the Improved Control Chart

The process is monitored by taking samples size and calculating the standard deviation using the mathematical formula expressed in equation 2 below. For each of all samples, the average value shown in the control chart is defined as follows:

$$\mu = \overline{\overline{X_s}} \approx \mu_0 = \frac{1}{m} \times \sum_{s=1}^{m} X_s \tag{1}$$

The associated standard deviation is given by:

$$\sigma_s = \frac{\sigma_0}{\sqrt{n}} \tag{2}$$

Upper control chart limits

$$\text{UCL} = \mu_0 + \frac{k_p}{\sqrt{n}} \times \sigma_0 \tag{3}$$

Lower control limit

$$\text{LCL} = \mu_0 - \frac{k_p}{\sqrt{n}} \times \sigma_0 \tag{4}$$

Upper Surveillance limit

$$\text{USL} = \mu_0 + \frac{k_{imp}}{\sqrt{n}} \times \sigma_0 \tag{5}$$

Lower Surveillance limit

$$\text{LSL} = \mu_0 - \frac{k_{imp}}{\sqrt{n}} \times \sigma_0 \tag{6}$$

The parameters of the chart are n, h, k_p, and k_{imp}. In this paper, we consider situations in which process property degradation may be related to problems related to machine degradation states, for example (Bouslah et al., 2016; Radhoui et al., 2010). Therefore, depending on the evolution of the quality indicator (X_s), it is decided whether and what kind of maintenance action should be performed. Following an inspection of sample size n, depending on the control limits, three scenarios are possible as shown in Figure 4, Figure 5, and Figure 6 below; Corrective maintenance without duration in the control state, imperfect maintenance in a surveillance state, and Perfect maintenance with duration (t_p) in the critical state.

Scenario I: As shown in Figure 4, when the production system is under control, the average of the measurements of the quality indicator $\bar{X}_s$ is located between the surveillance limits $LOL \leq \bar{X}_s \leq UOL$. The process is considered to be under control, and Corrective Maintenance actions with minimal repair are executed without any duration.

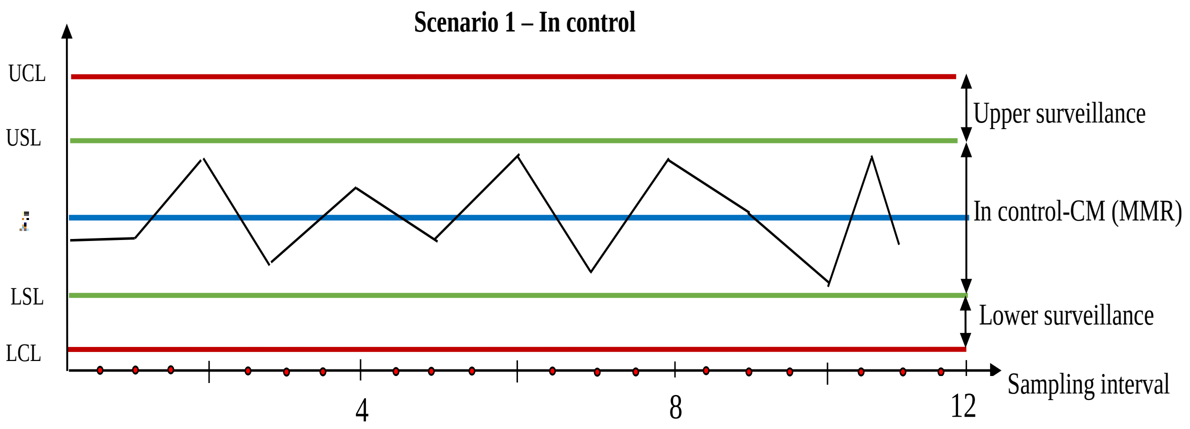

Figure 4. Control chart at scenario 1

Scenario II: As shown in Figure 5, When the production system shifted from the central limit to the surveillance state, the average of the measurements of the quality indicator $\bar{X}_s$ is located between the surveillance and the control limits $(LCL \leq \bar{X}_s \leq LSL \text{ or } USL \leq \bar{X}_s \leq$

UCL) for a sample j_{imp}. In this case, the process is considered to have entered a state called the monitoring state. Hence, the machine is briefly stopped and an imperfect preventive maintenance action with an average duration (t_{imp}) is carried out to mitigate the process and reduce the failure rate of the process. In this case, a small proportion of non-confirming units according to the short maintenance duration are detected and rejected. Assume that the number of non-conforming is proportional to the production volume between the sampling interval. So the measure of inspections m is therefore equal to j_{imp}.

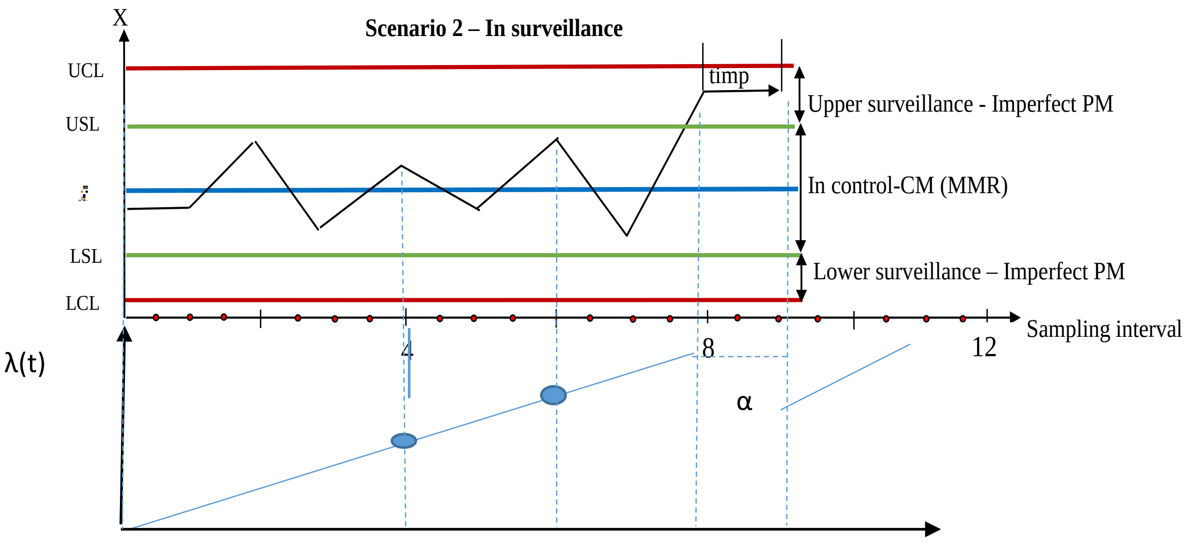

Figure 5. Case of an imperfect PM action triggered by the control chart

Scenario III: When the average of the quality indicator measurements is located beyond the control limits ($LCL \geq \bar{X}_s$ or $\bar{X}_s \geq UCL$) the machine is considered in an 'out-of-control' state called a critical state. And the production unit is considered in critical condition. In this case, the production is stopped and a Perfect maintenance action with a mean duration t_p is performed on it. All items produced between sampling intervals$(j_p - h)^{th}$, $(j_p)^{th}$are rejected. Therefore, the number of inspections m is equal to j_p.

Notations and Assumptions

Assumption

The study of this chapter is based on the following assumptions:

- Demands that are not satisfied at the end of each period are lost.
- The non-conformity of the products is due to the degradation of the production unit.
- Maintenance actions depend on the control chart parameters and the type of maintenance (perfect, imperfect, and with minimal repair).
- The resources needed to perform maintenance actions are available.
- The unit costs related to production, inventory, maintenance, and quality are known and constant.

Notations

Productions parameters

$u(k)$: Machine production rate $u = \{u\,(0), u(1), \ldots, u(H-1)\}$ u in period $k, (k = 0, 1, \ldots, H-1)$

$S(k)$: Principal store inventory at the period end ($k = 0, \ldots, H$)

S : Principal store S (Manufacturing Store).

$w_i(k)$: inventory level $w_i, (i = 0, .. L)$ at the end of the period $k, (k = 0, 1, \ldots, H-1)$

	for each retail store
τ_i	: Delivery time for retail stores R_i
$Q_i(k)$	: Delivery rate $Qi = \{Qi\ (0), Qi\ (1), \ldots, Qi\ (H-1)\}Ri$ at period $k, (k = 0, 1, \ldots, H-1)$ for the warehouses
L	: Number of retail stores.
Δt	: The production length
$d_i(k)$	: Average demand during the period $k, (k: 0, 1, \ldots, H-1)$ for each customer
$V_{di}(k)$	: Demand variance in period $k, (k: 0, 1, \ldots, H)$ for each customer
H	: The production planning horizon
$H.\Delta t$	: Finite production horizon
Q_v	: Capacity of the delivery vehicle.
c_p	: Machine unit production cost
c_{hS}	: The unit product cost of inventory holding during one period at the principal store.
c_{hwi}	: The cost of inventory holding unit product during one period at the retail store $R_i, (i = 0, .. L)$
c_d	: Shortage cost of the one-unit product during one period.
c_l	: the unit delivery cost
ct	: transportation cost of one item
mu	: Monetary unit.
u_{max}	: Maximum capacity of the production machine.
u_{min}	: Minimum capacity of the production machine.
θ_i	: Probability index related to each customer service level.

Maintenance parameters

$\lambda_k(t)$	: Failure rate at period k, (k = 1..., H)
N_{pef}	: Number of perfect maintenance actions during the out-of-control state (Scenario 3).
N_{imp}	: Number of imperfect maintenance actions during the surveillance state (Scenario 2).
t_p	: Duration of perfect maintenance
t_{imp}	: Duration of imperfect maintenance
C_{cm}	: Corrective maintenance cost
C_{Im}	: Imperfect maintenance cost
C_{pm}	: Perfect maintenance cost
C_{MT}	: Total maintenance cost

Quality parameters

h	: The sampling interval.
$\propto_2$	: The probability of non-identification of the surveillance state
$\propto_3$	: The probability of non-identification of the critical out-of-control state
ψ_1	: The parameter showing if the surveillance limits were exceeded before the control limits
j_p	: Average number of samples to detect the out of control
j_{imp}	: Average number of samples to detect the surveillance state
δ_p	: Magnitude of a shift in between SL to the Critical state
δ_{imp}	: Degree of a shift between the centre to the SL
k_p	: Control chart coefficient samples at a critical stage.

k_{imp}	: Control chart coefficient of samples at surveillance zone
ARC	: The average duration of a restoration cycle
c_{si}	: The unit sampling inspection cost
c_r	: The unit cost of one defective product
c_{NC}	: The cost of the non-conformal product
$A\ CQ$	: Average total cost of quality

MODEL OF THE STOCHASTIC PRODUCTION MAINTENANCE QUALITY PROBLEM

To establish an optimal production, maintenance, and quality integrated strategy, we define a stochastic model that minimizes the total costs over a finite horizon. The total expected cost includes the costs of production, inventory holding, delay penalties, and quality/maintenance.

Formally, the problem is defined as follows:

The optimization model is composed of objective functions that minimize the expected total costs (production, inventory, delivery, delay penalty, quality, and sum of maintenance) over the finite horizon $H.\Delta t$.

$$Min\ F = PC + HC + DC + DPC + TMC \tag{7}$$

Under the following constraints:

$$S(k) = S(k-1) + u(k)\cdot\Delta t - \sum_{i=1}^{L} Q_i(k) \quad \text{with } k = \{0, 1 \ldots, H\} \tag{8}$$

$$w_i(k) = w_i(k-1) + Q_i\left(k - {\tau_i}/{\Delta t}\right) - d_i(k)$$

$$\text{where } k, (k = 1, 2, \ldots, H)\ and\ \tau_i \geq 1 \tag{9}$$

The service level, for $(k = 1, \ldots, H-1)$ and $(i = 1, \ldots, L)$

$$Prob[w_i(k) \geq 0] \geq \theta_i \tag{10}$$

The upper and lower bounds of production, during each period

$$u_{min} \leq u(k) \leq u_{max} \tag{11}$$

To transform the probabilistic problem into a deterministic equivalent, it is necessary to first determine the change in the variance of inventory over the planning horizon.

Lemma:

For $k = 1,2, \ldots, H\ and\ i = 0, 1, \ldots, L$ we have;

$$Prob[w_i(k) \geq 0] \geq \theta_i \Rightarrow Q_i\left(k - {\tau_i}/{\Delta t}\right) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k-1) + \hat{d}_i(k) \tag{10.1}$$

With

φ: Cumulative Gaussian distribution function with mean $\hat{d}_i(k)$ and finite variance $V_{d,(k)}$.

$U_{\theta_i}(.)$: $Minimum\ production\ quantity$

at deliveryφ^{-1}: inverse distribution function.

Proof: We have, $w_i(k) = w_i(k-1) + Q_i\left(k - {\tau_i}/{\Delta t}\right) - d_i(k)$, and according to equation (10) we have $Prob[w_i(k) \geq 0] \geq \theta_i$, then the service level requirement constraint is given by:

$$Prob\left[w_i(k-1) + Q_i\left(k - {\tau_i}/{\Delta t}\right) - d_i(k) \geq 0\right] \geq \theta_i \tag{10.2}$$

We divide the expression by $\sqrt{V_{di}(k)}$, then we have:

$$Prob\left[\frac{w_i(k-1) + Q_i\left(k - {\tau_i}/{\Delta t}\right) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}} \geq \frac{d_i(k) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}}\right] \geq \theta_i \tag{10.3}$$

Note that:

$X = \frac{d_i(k)-\hat{d}_i(k)}{\sqrt{V_{di}(k)}}$ is a variable that follows the reduced centred Gaussian distribution N (0, 1)

The cumulative Gaussian distribution function is denoted by φ.

$$\varphi\left[\frac{w_i(k-1)+Q_i\left(k-{}^{\tau_i}/_{\Delta t}\right)-\hat{d}_i(k)}{\sqrt{V_{di}(k)}}\right] \geq \theta_i \quad (10.4)$$

Since $\lim_{-\infty} \varphi \to 0$ and $\lim_{-\infty} \varphi \to 1$ we conclude that φ is strictly increasing.

We note that φ is indefinitely differentiable, so we conclude that φ is invertible.

$$\frac{w_i(k-1)+Q_i\left(k-{}^{\tau_i}/_{\Delta t}\right)-\hat{d}_i(k)}{\sqrt{V_{di}(k)}} \geq \varphi^{-1}(\theta_i) \quad (10.5)$$

Then $w_i(k-1) + Q_i\left(k - {}^{\tau_i}/_{\Delta t}\right) - \hat{d}_i(k) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) Q_i\left(k - {}^{\tau_i}/_{\Delta t}\right) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k-1) + \hat{d}_i(k)$ (10.6)

Thus

$$Prob[w_i(k) \geq 0] \geq \theta_i \Rightarrow Q_i\left(k - {}^{\tau_i}/_{\Delta t}\right) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k-1) + \hat{d}_i(k) \quad (10.7)$$

Production, Maintenance, and Quality

Equipment availability, reliability, maintenance, product quality, and System Productivity are strongly interrelated to each other.

Production policy

In this section, we aimed to minimize the total production and inventory cost by determining the optimal and economical production planning to meet the random demands of the customers at multi-warehouses and throughout the production period.

Recall that the total production Cost (TCP):

TCP = Production cost + Total holding cost +
+ Delivery cost + Delay penalty cost.

- **Production Cost**

$$PC = C_p \times \sum_{k=1}^{H} u(k).\Delta t \quad (13)$$

- **Holding cost at the Manufacturing Store**

$$HC_S = \sum_{k=1}^{H} C_{hS} \times \left(Max(S(k-1),0).\Delta t + \tfrac{1}{2}.u(k).\Delta t^2\right) \quad (14)$$

- **Holding costs at the retail warehouses (w_i)**

$$HC_W = \sum_{k=1}^{H} \sum_{i=1}^{L} C_{hwi} \times \left(Max(w_i(k-1),0).\Delta t + \tfrac{1}{2} Q_i\left(k - {}^{\tau_i}/_{\Delta t}\right).\Delta^2\right) \quad (15)$$

- **Total Inventory Holding cost (HC)**

$HC = HC_S + HC_W$

$$HC = \sum_{k=1}^{H}\left(C_{hS} \times \left(Max(S(k-1),0).\Delta t + \tfrac{1}{2}.u(k).\Delta t^2\right) + \sum_{i=1}^{L} C_{hwi} \times \left(Max(w_i(k-1),0).\Delta t + \tfrac{1}{2} Q_i\left(k - {}^{\tau_i}/_{\Delta t}\right).\Delta^2\right)\right) \quad (16)$$

- **Delivery cost**

$$DLC(Q(k)) = C_t + C_o \times \sum_{i=1}^{L}\left(\frac{Q_i(k)}{Q_v}\right) \quad (17)$$

- **Delay Penalties cost**

$$DPC = C_d \times \left(\sum_{k=1}^{H}\left(\sum_{i=1}^{L} dw_i\right)\right), \quad dw_i = \frac{|\min(w_i(k),0)|}{Q(k+1-\frac{\tau_i}{\Delta t})} \quad (18)$$

Maintenance and quality policy

The optimization of maintenance strategy consists to minimize the costs related to perfect (critical state) and imperfect (surveillance state) preventive maintenance actions and corrective maintenance actions (in-control state).

Failure rate modelling

$$\lambda_k(t) = \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) \times j_{imp}} \right\rfloor\right) \times \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p} \right\rfloor\right) \times \left(\lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t)\right) + \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p} \right\rfloor\right) \times \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) j_{imp}} \right\rfloor \times \lambda_{\left(\mathrm{k} - \left\lfloor \frac{j_{imp}}{\Delta t} \right\rfloor\right)} \left(\left(\left\lfloor \frac{k \times \Delta t}{j_{imp}} \right\rfloor - 1\right) \times j_{imp}\right) \times e^{\alpha} + \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p} \right\rfloor \times \frac{u(k)}{U_{max}} \lambda_n(t) \quad \forall\, t \in [0, \Delta t]$$

(19)

The average total maintenance cost

Considering that either PM (perfect or incomplete) or CM actions can be performed in each cycle, the total average maintenance cost can be expressed as:

$$C_{TM}(j_p, j_{imp}) = C_{cm} * (1 - \psi_1) + CM_{imp} * \psi_1 + CM_{perf} * \varphi(j_p, j_{imp}) \tag{20}$$

The average number of failures $\varphi(j_p, j_{imp})$ is expressed as follows:

$$\varphi(j_p, j_{imp}) = \sum_{k=1}^{\left\lceil \frac{ADRC - (t_{imp} \times \psi_1 + t_p \times (1-\psi_1))}{\Delta t} \right\rceil} \int_0^{\Delta t} \lambda_k(t)\, dt + \sum_{k=\left\lfloor \frac{ADRC}{\Delta t} \right\rfloor}^{H} \int_0^{\Delta t} \lambda_k(t)\, dt$$

(21)

Average duration of a restoration cycle

Consider an average number of samples to signal the process moves into the surveillance state. From Montgomery (2004) it follows that;

$$j_{imp} = \frac{1}{1 - \alpha_2}$$

$$\alpha_2 = \mathrm{F}\left(\frac{USL - \mu_1}{\sigma/\sqrt{n}}\right) - \mathrm{F}\left(\frac{LSL - \mu_1}{\sigma/\sqrt{n}}\right) \tag{22}$$

Hence:

$$\alpha_2 = \mathrm{F}\,(k_{imp} - \delta_{imp} \times \sqrt{n}) - \mathrm{F}\,(-k_{imp} - \delta_{imp} \times \sqrt{n})$$

(23)

Using Equation (3.23), we obtain:

$$j_{imp} = \frac{1}{1 - \mathrm{F}\left(k_{imp} - \delta_{imp} \times \sqrt{n}\right) + \mathrm{F}\,(-k_{imp} - \delta_{imp} \times \sqrt{n})}$$

(24)

Let j_p be the average number of samples to signal the shift to the critical out-of-control state. It also follows that;

$$j_p = \frac{1}{1 - \alpha_3} \tag{25}$$

$$\alpha_3 = \mathrm{Prob}(\, LCL \le \bar{X}_s \le UCL / \mu_2 = \mu + \delta_p \times \sigma_s) \tag{26}$$

$$\alpha_3 = \mathrm{F}\left(\frac{UCL - \mu_2}{\sigma/\sqrt{n}}\right) - \mathrm{F}\left(\frac{LCL - \mu_2}{\sigma/\sqrt{n}}\right)$$

$$\alpha_3 = \mathrm{F}\,(k_p - \delta_p \times \sqrt{n}) - \mathrm{F}\,(-k_p - \delta_p \times \sqrt{n}) \tag{27}$$

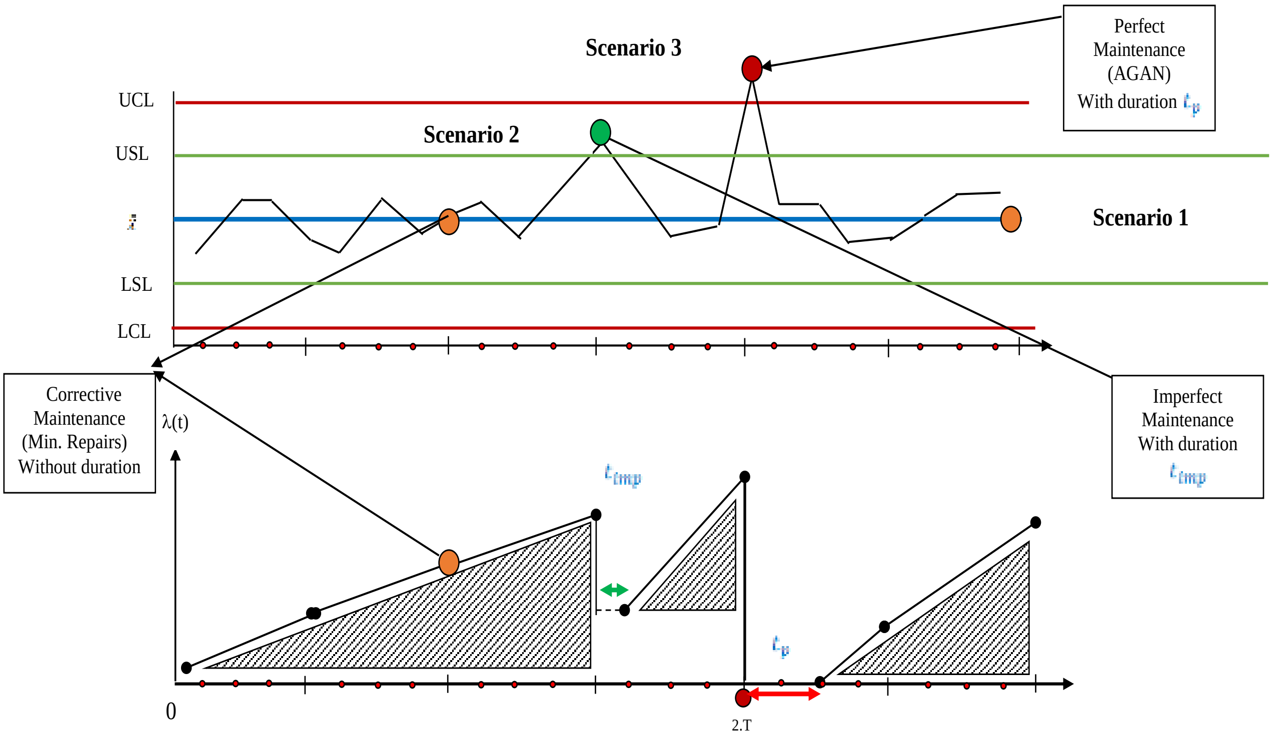

Figure 6. Integrated maintenance and control chart decisions

$$j_p = \frac{1}{1-\mathrm{F}\left(k_p-\delta_p\times\sqrt{n}\right)+\mathrm{F}\left(-k_p-\delta_p\times\sqrt{n}\right)} \quad (28)$$

The average duration of a restoration cycle:

$$\text{ADRC}= \left(j_{imp} \times \psi_1 + j_p \times (1-\psi_1)\right) \times h + \mu_{imp} \times \psi_1 + \mu_p \times (1-\psi_1) \quad (29)$$

ψ1 is the parameter showing if the surveillance limits were exceeded before the control limits. This indicator is defined by the following expression;

$$\psi_1 = \text{mark}\,(j_{imp} < j_p) = = \begin{cases} 1, & If\ j_{imp} < j_p \\ 0, & Otherwise \end{cases} \quad (30)$$

The average overall cost of quality

The average total cost of quality is the sum of sampling inspection plus the cost of non-conformal.

The sampling inspection is expressed by;

$$C_{si} = C_i \times n \times m \quad (31)$$

The cost of non-conforming units.

$$\text{CNC} = C_r \times \left(\psi_1 \times \left(u\left(\left\lfloor\frac{j_p*h}{\Delta t}\right\rfloor\right) \times \left(\frac{j_p*h}{\Delta t} - h\right)\right) + (1-\psi_1) \times \left(u\left(\left\lfloor\frac{j_{imp}*h}{\Delta t}\right\rfloor\right) \times \left(\frac{j_{imp}*h}{\Delta t} - h\right)\right)\right)$$

OPTIMIZATION MODEL

A sequential optimization procedure is used. Solving the first sub-problem of production and its output is considered as a constraint to the maintenance quality sub-problem. An iterative numerical optimization procedure is developed. The procedure integrated the following user inputs.

- The multi-decision variables (n, m, h, k_p, k_{imp}) are initialized with small values by the user.
- The decision parameters are variated with some increments $(\Delta n, \Delta m, \Delta h, \Delta k_p, \Delta k_{imp})$)

to some defined maximum limits ($n_{max}, m_{max}, h_{max}, k_{p_{max}}, k_{imp_{max}}$)

- The average number of checks (of successive samples) to be in or out of control (j_{imp}, j_p) is calculated by using equations (21 & 25)
- The average duration restoration cycle (ADRC) is calculated by equation (29)
- The total cost of maintenance is calculated using equation (20).

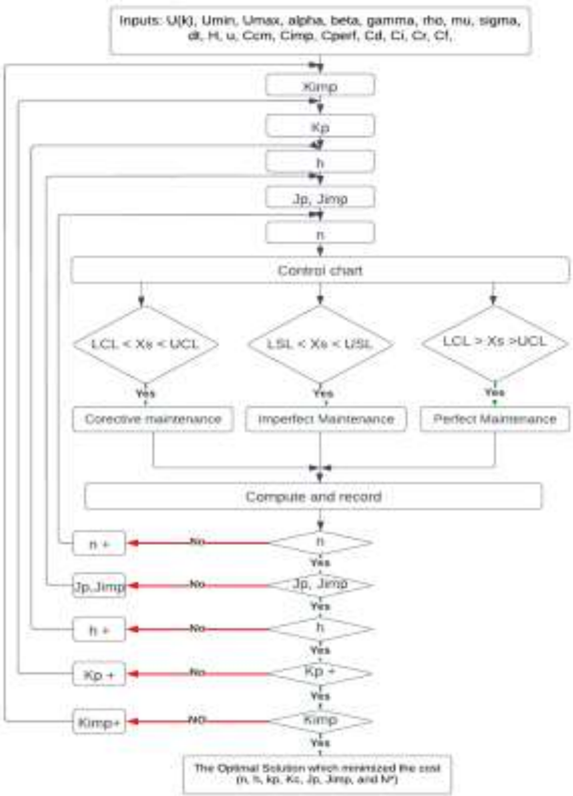

Figure 8. Maintenance & Quality algorithm

NUMERICAL EXPERIMENT

A Supply Chain consists of a system producing single product to meet the delivery of two retail stores that will satisfy random demands within a finite production periods of length one month. The standard deviation of each product demand is the same for all periods and each demand. Lower and upper boundaries of production, as well as other production, maintenance, and quality parameters presented below were used for experiments with our model as an approach to finding the best strategy.

Input data

Production		**Maintenance**		**Quality**	
PARAMETER	*VALUE*	*PARAMETER*	*VALUE*	*PARAMETER*	*VALUE*
u_{max}	5000	β_1	100	Ci	50
u_{min}	0	α_1	2	Cr	70
$S(0)$	0	CM_{imp}	500	μ_0	5
τ_i	1.0	CM_{perf}	1000	σ_0	1.5
L	2.0	C_{cm}	70000	δimp	0.8
Δt	1.0	t_p	4 hrs	δp	1.0
$d_i(k)$	1500				

H	12
Q_v	3000
c_p	50
c_{hS}	2.2
c_{hwi}	2.2
c_d	4.0
τ_i	1.0
θ_i	0.9

Results

The optimal production quantity Table 1. The production plans are according to the probabilistic function defined by equation (10).

Table 1. Optimal Production Plan

u(1)*	*u*(2)*	*u*(3)*	*u*(4)*	*u*(5)*	*u*(6)*
4540	2940	2530	3220	3210	4200
u*(7)	u*(8)	u*(9)	u*(10)	u*(11)	u*(12)
4110	4220	3630	2430	2540	4643

Table 2. The delivery plan for a retail house 1

Q_1-1	Q_1-2	Q_1-3	Q_1-4	Q_1-5	Q_1-6
2250	2150	2050	2500	1250	2040
Q_1-7	Q_1-8	Q_1-9	Q_1-10	Q_1-11	-
1340	1280	2150	1080	1010	-

Table 3. The delivery plan for a retail house 2

Q_2-1	Q_2-2	Q_2-3	Q_2-4	Q_2-5	Q_2-6
2150	640	1130	2250	1210	1540
Q_2-7	Q_2-8	Q_2-9	Q_2-10	Q_2-11	-
470	320	2100	1130	2350	-

The detailed algorithm flowchart diagram is shown above in Figure 8.

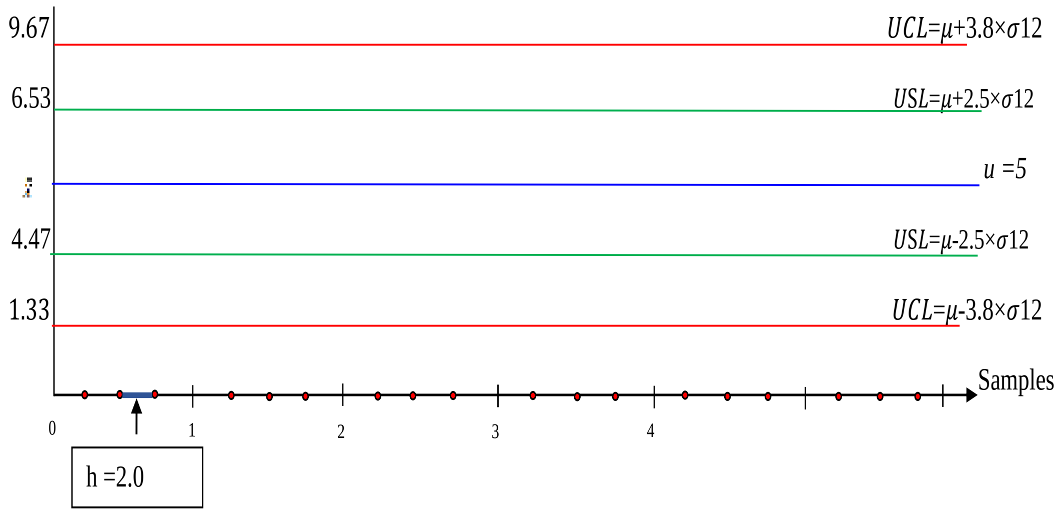

Figure 9. Optimal design of the control chart

Table 4. Optimal decision parameters

h	n	m	k_{imp}	k_p	J_pT	$J_{imp}T$	N_{imp}
3	48	57	2.50	3.5	2928	54	2

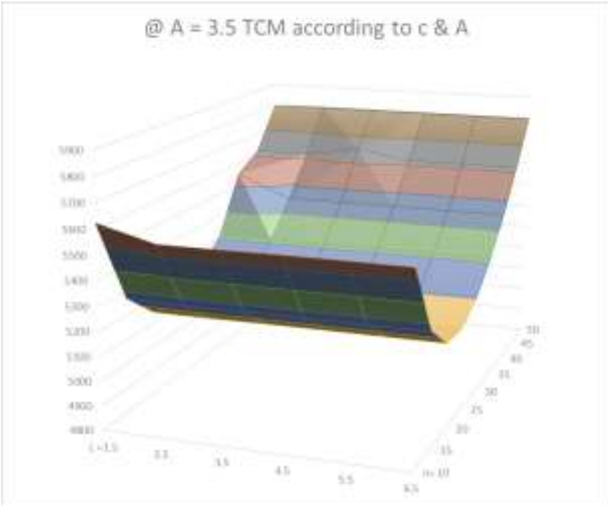

Figure 10. The total cost of maintenance against control chart combinations

Discussion of the Results

Table 1 presents the optimal planned production quantities. The optimal delivery quantities at the multi-retail store are presented in Tables 2 and 3 which guarantee the customer service level requirement. The best maintenance and quality strategy consist in considering one sample of size 48 every h, with h = 3 days. Furthermore, concerning the scheme of the control chart, the ideal number of standard deviations between the centerline, the surveillance limits, and the control limits are respectively 2.50 and 3.50. The shift to the surveillance state and out-of-control state would occur on average after 54h and 2928h respectively. The optimal parameters of the control chart are shown in Table 4 which minimizes the total cost of maintenance and quality.

Whenever a sample is considered, the percentage that the system is in the 'in-control' state is found to be equal to 74%, the probability to be in the surveillance state (performing an imperfect PM action) is 18%, and the probability to be in the critical state and perform a perfect maintenance action is 8%.

Sensitivity Analysis for Maintenance/Quality Strategy

In this subsection, we investigate the influence of changing some key study parameters. To analyse and validate the system's behaviour, we have studied the different possible causes of variations in model parameters, namely: the cost of inspection (c_i), the cost of rejecting defective items (c_r), imperfect maintenance duration (t_{imp}), and the magnitude of the transition to the surveillance state and the critical "out-of-control" state relative to the centerline (δ_{imp}). The effects of variations in these parameters on the optimum solutions are given in Table 5. Figure 10 illustrated the effects of control chart combinations on the total maintenance cost. Additionally, we have studied and analysed the impacts of the different costs of maintenance imperfect (CM_{imp}).

From the sensitivity analysis (Table 6) presented below, we can deduce the following:

Variation of the unit inspection cost (C_i): The increase in unit inspection cost causes a decrease in the number of inspection items size (n) to optimize the overall cost of sampling, then a slight increase several times to take the sampling (m), to the total sampling expenditures.

Moreover, to reduce the frequency of sampling, the sampling interval h^* increases. This makes both surveillance and control limits coefficients ${k_p}^*$, ${k_c}^*$ increase, thereby yielding lower non-conformal products.

Variation of the cost of a defective unit (C_r): The increase in the rejection cost mainly causes a decrease in the sampling size ($\boldsymbol{n}$). And an increase in the times the sampling inspection is conducted ($\boldsymbol{m}$), And the increase in the control chart limit ($\boldsymbol{k_p}$) to reduce the proportion of non-conforming units. However, the cost of rejection affects the average number of samples to detect the surveillance state duration ($J_{imp}T*$) without significance on the average number of samples to detect the "out of control" (J_pT*). The surveillance and the control limits get closer, when the unit rejection cost $\mathrm{C_r}$ takes a tall, permitting possibly more focuses to drop near to them. Moreover, the optimal interval of sampling diminishes and the optimal sampling size increases to quickly detect small process shifts and reduce the proportion and the cost of non-conforming items. In like manner, the average numbers of samples (j_p and j_c) controlled before the shift to the surveillance or critical state diminishes, which illustrates the truth that maintenance activities are performed greater.

Variation of Imperfect Maintenance duration at out-of-control (t_{imp}): As the PM duration μ_{imp} gain, the coefficients of the control chart boost, the surveillance and control limits move further apart, and PM actions are done less frequently. After that, the average number of samples taken (J_{imp}) before going into the surveillance or critical state increase. Control charts with lower severity have higher sampling intervals and lower sample sizes. Subsequently, the quantity of non-conforming items hikes, increasing the costs of rejection.

Variation of the magnitude of the shift to the surveillance state (δ_{imp}): The increasing of the parameter (δ_{imp}) implies that the average sample of the quality indicator to detect the surveillance state ($J_{imp}T$) will move away from the centre line of the control chart. Hence, the control chart becomes more relaxed to cope with the increasing process degradation, allowing potentially more points to fall within the control limits. Consequently, the perfect maintenance actions with duration decrease as well as the average number of samples to detect the surveillance state ($J_{imp}T$). Furthermore, the sample size (n), decreases, and the sampling frequency increase thereby improving the process quality. As the parameter δp increases, the process deteriorates further and the new mean of the process moves away from the target mean μ. As a result, the monitoring limits have shorter coefficients and cramped charts. The result is a decrease in the average number of samples exceeding the monitoring limit and holding the frequency of maintenance activities. Other points are very close to the surveillance limits and between the surveillance and the control limits. A process may not run properly.

Table 5. Sensitivity analysis of parameters

Parameter	Variation	k_p	K_{imp}	h*	n*	m*	J_pT	$J_{imp}T$	N_{imp}
Base		3.85	2.5	3	48	57	2928	54	2
c_i	-5.00	3.85	2.5	3	48	57	2928	57	2
	0	3.95	2.5	3	36	57	3784	68	2
	+5.00	3.85	2.5	3	22	65	2928	74	2
c_r	-20.0	3.85	2.5	3	48	57	2928	54	2
	0	3.90	2.5	3	46	60	2928	54	2
	+20.0	3.95	2.5	3	32	65	3276	74	2
t_{imp}	-0.50	3.95	2.5	3	38	72	2964	78	2
	0	3.95	2.5	3	46	75	2964	78	2
	+0.50	3.85	2.5	3	48	54	2928	54	2
	-0.1	3.95	2.5	3	36	58	3588	32	2

δ_{imp}	0	3.95	2.5	3	48	57	3714	68	2
	+0.1	3.85	2.5	3	36	58	2806	32	2
cm-ip	-100	3.95	2.5	3	48	57	3744	68	2
	0	3.95	2.5	3	44	62	3432	72	2
	+100	3.95	2.5	3	44	62	3432	72	2

Therefore, the sample size is increased the sampling interval is decreased, and the quality of the process is improved.

Variation of Cost of Preventive Maintenance imperfect (CM_{imper}): The CM_{imper} which increases the cost of an IPM action, leads to the less frequent execution of IPM actions (higher value of jimp). In this case, the surveillance and the control limits are moving apart. So potentially more points fall within the limits of the monitor, and fewer points are very close to those limits. This accounts for the fact that the optimal sample size decreases and the sampling interval increases.

CONCLUSION

Most of the scientific work in the literature uses constant production plans, safety stocks, or constant failure rate methods. In this work, we optimized stochastic production planning and maintenance integrated into the control chart tool of quality. We considered the impact of the production variation on the formulation of a new failure rate model according to an increasing degradation factor. During this study, we elaborated an improved statistical process chart. We model and simulate an integrated production model using the ISPC tool that was used to control maintenance and quality. The production process reliability as well as the quality of the finished product are improved. This technique saves the company time and cost. A mathematical model and numerical procedure that significantly reduces the total cost of production, maintenance, inventory, and quality was established. The approach provided the control chart information based on established analytical relationships and significantly increases the process reliability, reduces the generation of non-conformal items, and minimised the total cost. The new maintenance approach considered the deployment of different types of maintenance with appropriate durations commensurate to multi-process scenarios.

Imperfect and perfect preventive maintenance actions are respectively performed according to an average number of samples to detect the surveillance and critical limits. However, depending on the production rate and dynamic inventory plan a corrective minimal repair is adapted in the control state to reduce the failure frequency. From the numerical experiments and the sensitivity analysis, this is found to be a robust approach, applicable to industrial cases in need of high precision principles required for the production of high-quality products. In perspective, the subsequent works will take into account considerable causes or multi-product cases.

AVAILABILITY OF DATA AND MATERIAL

Authors declare the presence of data associated with the manuscript, whether in repositories, available upon request, included in supplementary information, or in the figure source files.

CODE AVAILABILITY

We declare the availability of the application software and the programming custom code.

DECLARATIONS

Consent to participate: Informed consent was obtained from all individual participants included in the study.

Consent for publication: The participants have consented to the submission of the case manuscript to the journal.

COMPETING INTERESTS

We declare here that no conflicts of interest to disclose.

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AUTHORS CONTRIBUTION

Conceptualization and Writing of the original draft, Aminu Sahabi ABUBAKAR; methodology, Aime C. NYOUNGUE; Software, visualisation and analysis, Zied HAJJEJ, and Aminu Sahabi ABUBAKAR; editing, Zied HAJJEJ and Aime C. NYOUNGUE.

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